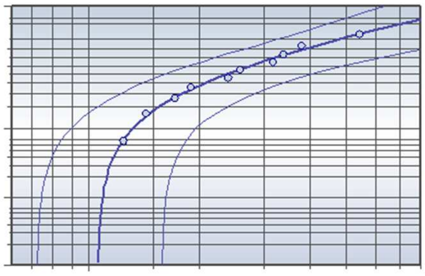
Handbook

of Weibull & Reliability Methods



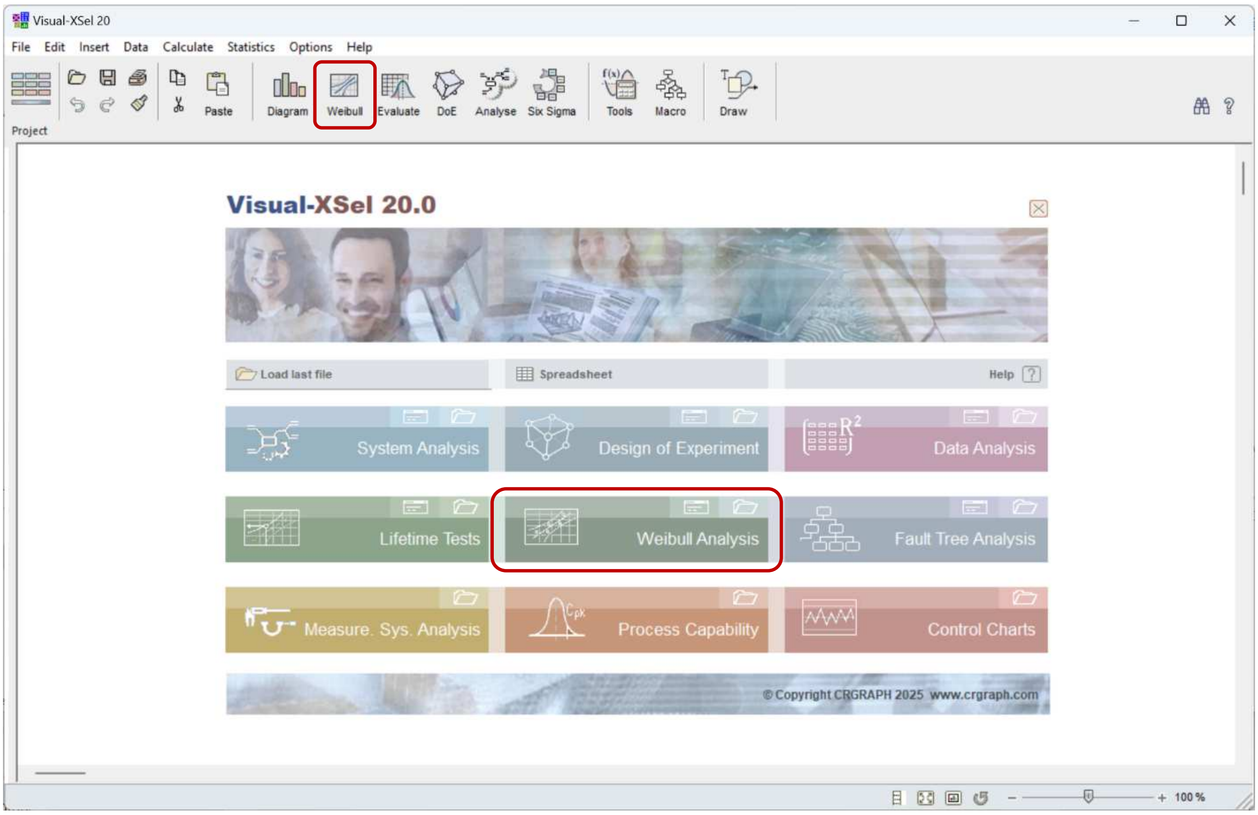
The most important
Methods and procedures
for practice

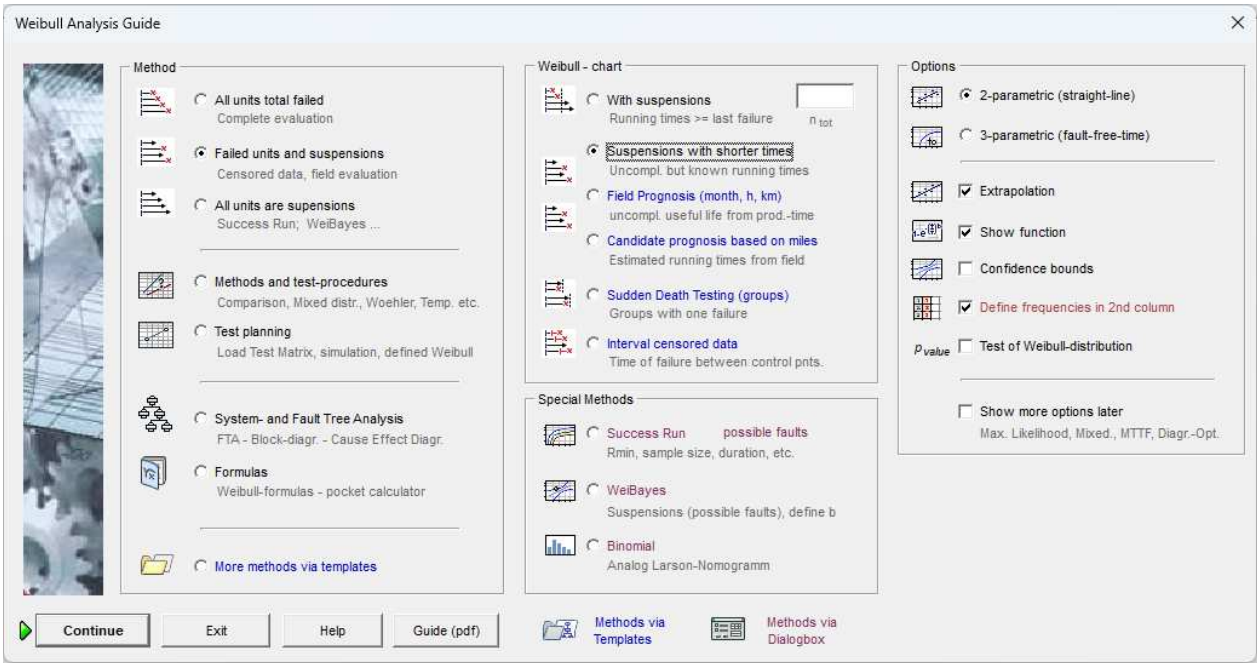
Curt Ronniger

www.weibull.de

1. Software Visual-XSel

All of the methods in this book can be performed with the software **Visual-XSel**





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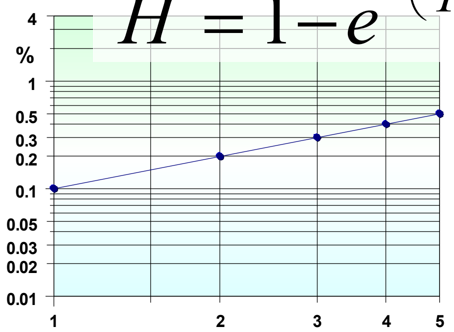
2. Introduction

Reliability is

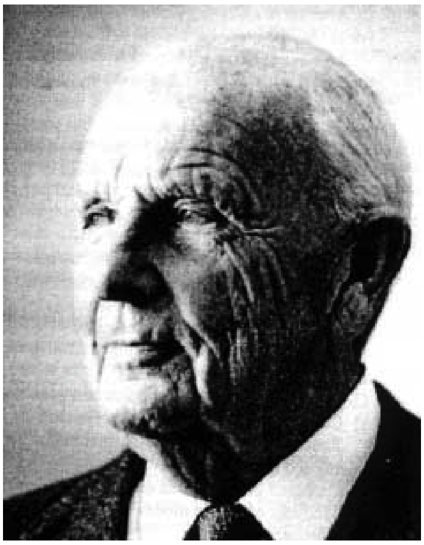
- ⇨ when a product does not fail
- ⇨ when a product is not impaired in terms of its function
- ⇨ when an expected lifetime is reached
- ⇨ when a produst satisfies expectations
- ⇨ quality

$$H = 1 - e^{-\left(\frac{t}{T}\right)^{b}}$$



Mathematically, the statistical fundamentals of **Weibull** and the associated distribution in particular are used to define reliability and unreliability.
This distribution was named by Waloddi Weibull who developed it in 1937 and published it for the first time in 1951. He placed particular emphasis on the versatility of the distribution and described 7 examples (life of steel components or distribution of physical height of the British population).
Today, the Weibull distribution is also used in such applications as determining the distribution of wind speeds in the design layout of wind power stations.
The then publication of the Weibull distribution was disputed – today it is a recognised industrial standard.

This study concerns itself with statistical methods, especially those formulated by Weibull. The Weibull analysis is the classic reliability analysis or the classic life data diagram and is of exceptional significance in the automobile industry. The "characteristic life" as well as a defined "failure probability" of certain components can be derived from the so-called Weibull plot.



Waloddi Weibull 1887 - 1979

It is proven to be of advantage to assume the cumulative distribution of failures as the basis for calculations. The distribution form used in the Weibull calculation is especially suited to this field of application. In general terms, the Weibull distribution is derived through exponential distribution. Calculations are executed in this way because:

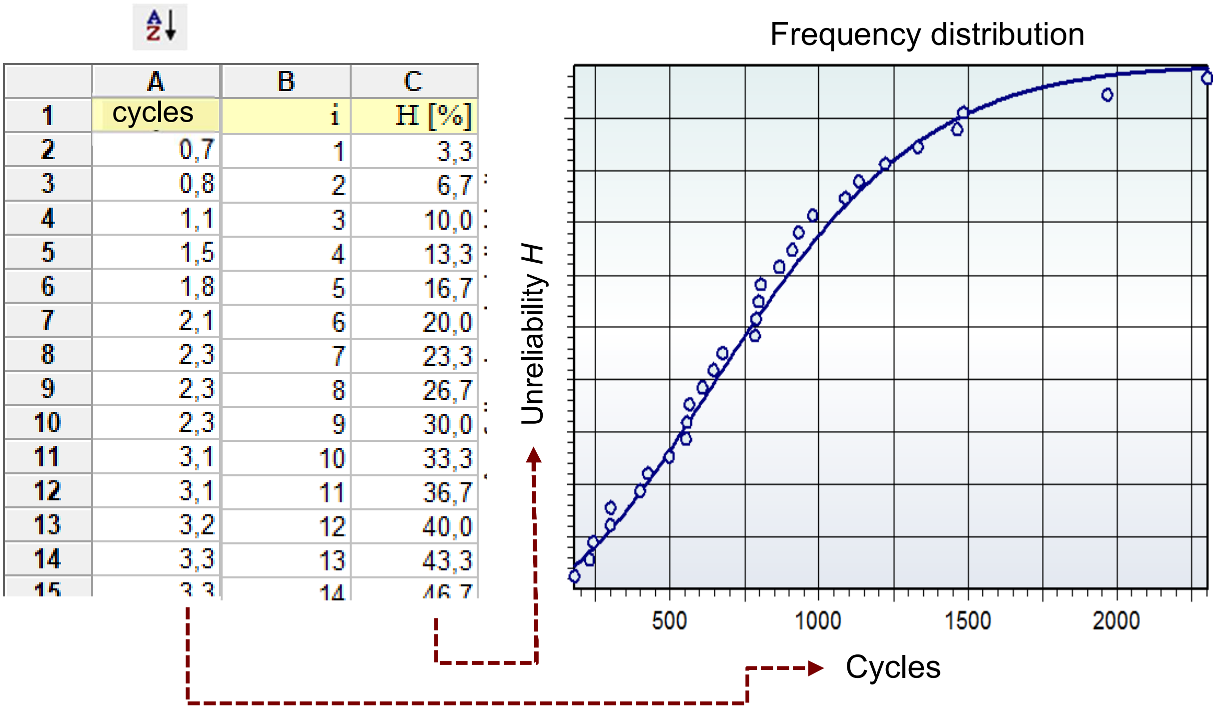
- Many forms of distribution can be represented through the Weibull distribution
- In mathematical terms, the Weibull functions are user-friendly
- Time-dependent failure mechanisms are depicted on a line diagram
- The method has proven itself to be reliable in practical applications

The methods and calculations discussed in this study are based on the corresponding VDA® standard and extend to practical problem solutions based on realistic examples.

Various methods (discussed in detail in this study) are used for the purpose of determining the parameters of the Weibull functions. Mathematical methods of deriving the parameters are generally not used in the majority of cases. Reference is therefore made to the corresponding specialised literature.

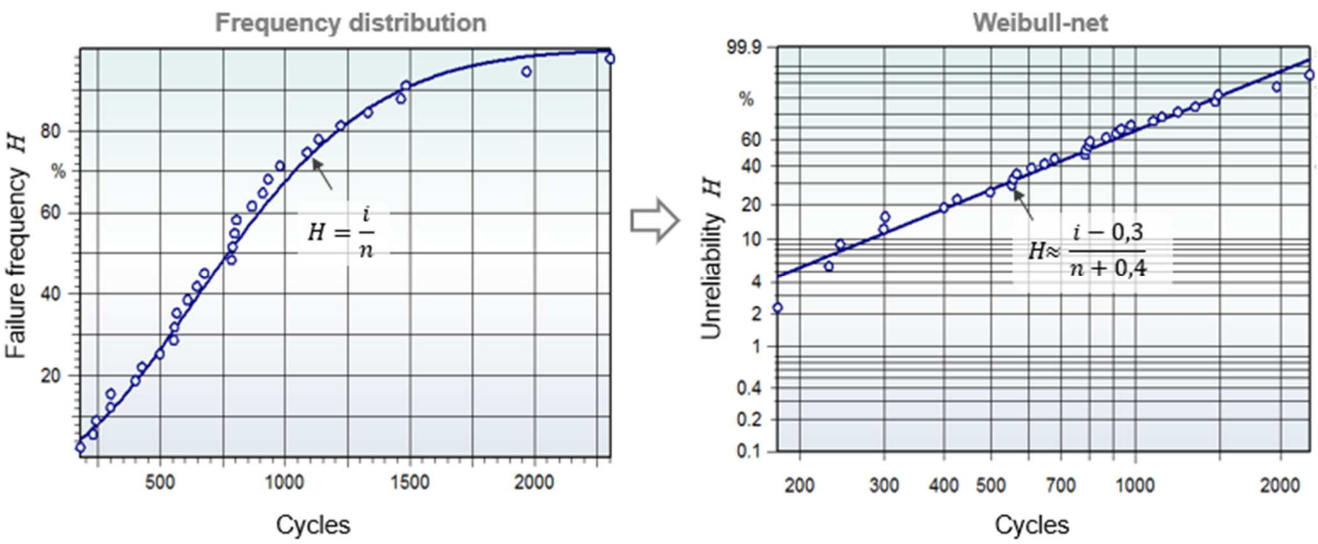
3. Basics for a lifetime analysis

This is a series of tests with n = 30 ball bearings, which are "driven" up to a test bench.
The cycles are initially carried out in a sheet entered, sorted in ascending order.
The frequencies are then $H = i / n$



	A	B	C
1	cycles	i	H [%]
2	0,7	1	3,3
3	0,8	2	6,7
4	1,1	3	10,0
5	1,5	4	13,3
6	1,8	5	16,7
7	2,1	6	20,0
8	2,3	7	23,3
9	2,3	8	26,7
10	2,3	9	30,0
11	3,1	10	33,3
12	3,1	11	36,7
13	3,2	12	40,0
14	3,3	13	43,3
15	3,3	14	46,7

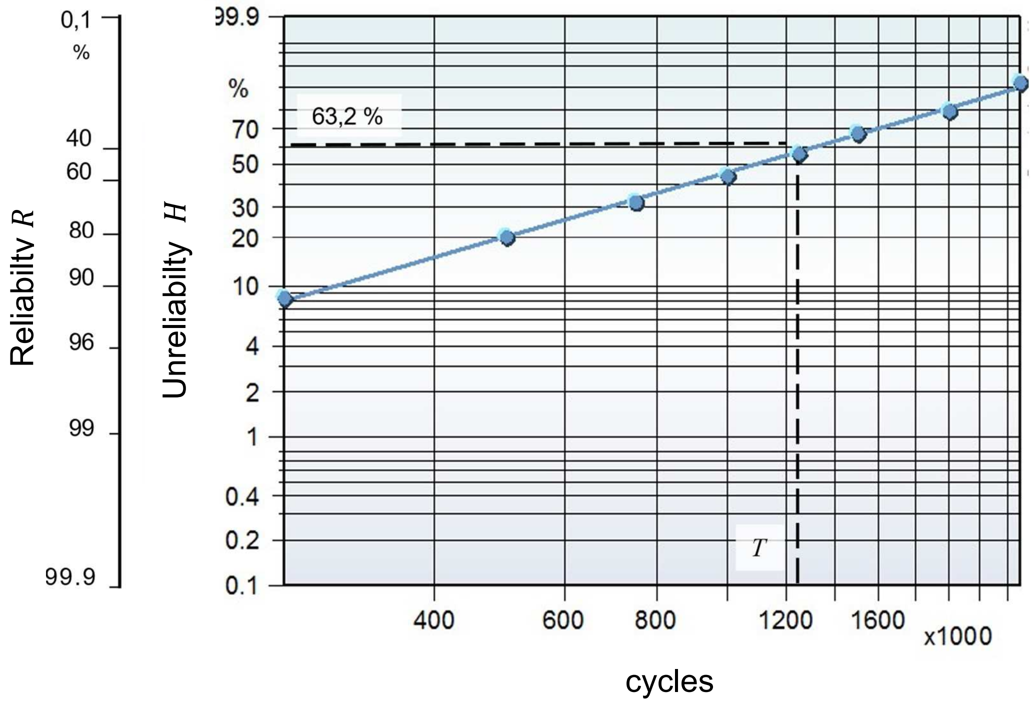
By multiple logarithmic axes you get a straight line. These axes scaling is not possible in MS-Excel. This representation is also called the Weibull net. The frequencies are calculated here using a correction formula, since one assumes a sample in which one does not reach the 100% of the population at $i = n$. Here the frequency is therefore $H=(i-0,3)/(n+0,4)$



The 2-parametric Weibull probability net

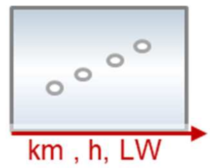
$R = 1 - H$

$R = e^{-\left(\frac{t}{T}\right)^b}$

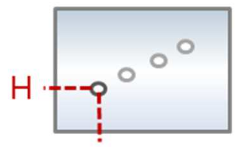


Building up the Weibull net

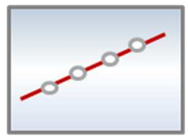
- **Step 0: Data preparation and relevant unit.**
  Where does the data come from and what is the correct reference?



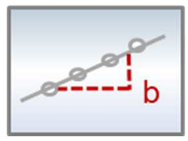
- **Step 1: Determining failure frequencies.**
  The information of the running times is given (X position). Determination of the cumulative frequencies of the points (Y- direction).



- **Step 2: The Weibull function & parameters**
  How to draw the straight line through the points.
  Calculation of the Weibull parameter
  ⇨ Weibull function



- **Step 3: Interpretation of the Weibull parameters.**
  Which information tell us the slope parameter b



Step 0: Data preparation and relevant unit

Generally, the reliability of components, units and vehicles can be determined only when failures occur, i.e. when the service life of the units under observation is reached. It is first necessary to verify the service life, e.g. by way of testing, in the laboratory or in the field, in order to be able to make a statement concerning or deduce the reliability.

Life characteristic

In the majority of cases, the life characteristic or lifetime variable t is a
- driven distance
- operating time
- operating frequency
- number of stress cycles

One of these data items relating to the "defective parts" to be analysed must be available and represents the abscissa in the Weibull plot.

Classification

For a random sample of n>50, the failures are normally classified such as to combine certain lifetime ranges. Classification normally results in a more even progression of the "Weibull curve". The classification width can be estimated in accordance with Sturges with

$$K_{br} = \frac{1}{1 + 3{,}32\,\lg(n)}$$

In practical applications, the class width or range, especially for field data involving kilometre values, is appropriately rounded up or down to whole thousands, e.g. 1000 km, 2000 km, 5000 km etc. In the frequency distribution (density function), the classes are assigned midway between 500 $\leq X <$ 1500 km. In the cumulative distribution, however, the classes must be referred to the upper classification limit: 1..1000, 1001..2000, 2001..3000 are assigned to the classes 1000, 2000 and 3000 km etc.

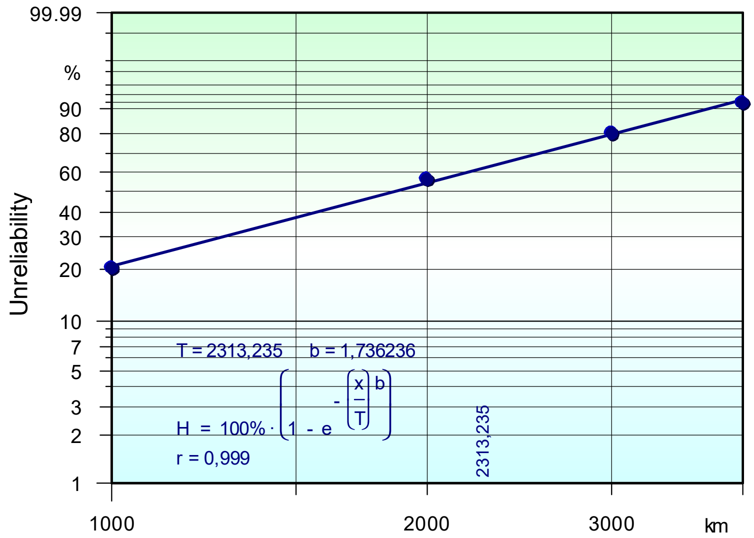
Inevitably, data are lost through classification, resulting in slight deviations for different classifications when calculating the Weibull parameters. The same procedure or the same classification should therefore always be chosen when comparing different analyses.

Multiple failures

It is important to note that in the case of "multiple failures" for which classes are defined, the result is not the same as when each failure is specified individually one after the other. For example: both tables represent the same circumstances, the first set of data is classified the lower set is listed as individual values:

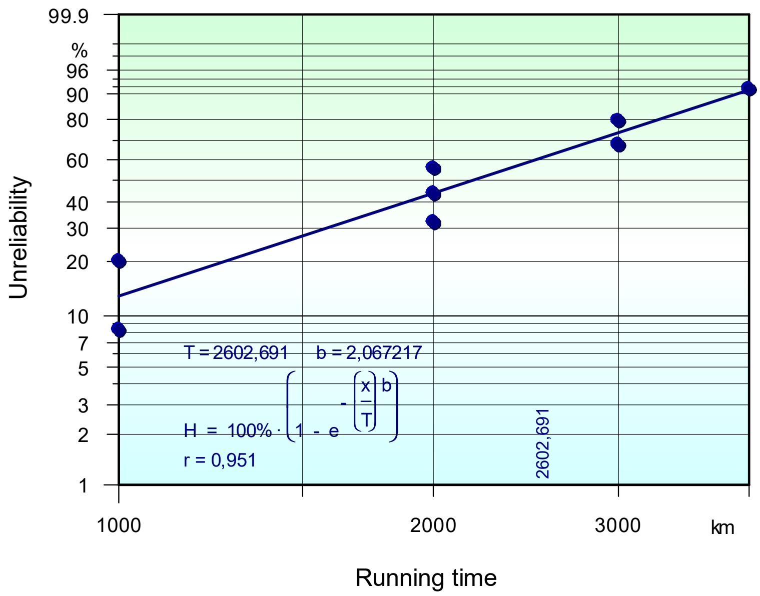
Classified data

Lifetime	Quantity
1000	2
2000	3
3000	2
4000	1



Individual values

Lifetime	Quantity
1000	1
1000	1
2000	1
2000	1
2000	1
3000	1
3000	1
4000	1



When represented in the Weibull plot as a best-fitting straight line, there are differences in the Weibull parameters attributed to the point distribution in the linearised scale. Although the classification is therefore not incorrect, it is recommended as from a quantity of more than 50 data items.

0-running time failures

Parts which are defective before being put to use are to be taken out of the evaluation. These parts are known as “0-km failures”. Added to this, points with the value 0 are not possible in the logarithmic representation of the X-axis in the Weibull plot. There is also the question of how failures are counted that have a distance rating of 50, 100 or 500 km as these failures also attributed to a defect or any other reasons. Particular care must be taken when defining the classification to ensure that mathematically the distance covered (mileage) is set to 0 between 0 and the next classification limit depending on the width of the classification range. The number of these "0-km failures" is to be specified in the evaluation.

General evaluation problems

If it is necessary to analyse failed components that were already in use (so-called in-field failures), the failure probability can be calculated using the previously described methods. A defined production quantity n is observed for a defined production period and the number of failures is calculated from this quantity.

Incorrect findings

The prerequisite is, of course, that all failures of this production quantity have been recorded and that there are not incorrect findings. Incorrect findings relate to components that are removed and replaced due to a malfunction but were not the cause of the problem. These parts are not defective and therefore also did not fail. For this reason they must be excluded from the analysis. Added to this, it is also important to take into account the life characteristic. Components that have been damaged due to other influences (due to an accident) for example) should not be included in the analysis. **Damage analysis** must therefore always be performed prior to the actual data analysis.

Multiple complaints

Parts already replaced in a vehicle must also be taken into account. If a replaced component is renewed, it will have a shorter operating performance rating in the vehicle than indicated by the kilometre reading (milometer). An indication that components have already been replaced are vehicle identification numbers occurring in double or several times in the list of problem vehicles. The differences in the kilometre readings (mileage) should then be used for the evaluation (please refer to *Repeatedly failed components*).

Step 1: Determining the failure frequencies

By sorting all defective parts in ascending order according to their life characteristic, the corresponding failure probability H can already be determined in very simple form with the following approximation formula:

$$H = \frac{i - 0.3}{n + 0.4} 100\%$$

and if there are several failures classified:

$$H = \frac{G_i - 0.3}{n + 0.4} 100\%$$

were

i : Ordinal for sorted defective parts

G_i : Cumulative number of cases

n : Reference quantity, e.g. production quantity

For n ≥ 50 one counts often also on the easy formula:

$$H = \frac{i}{n + 1} 100\%$$

or using the classified version:

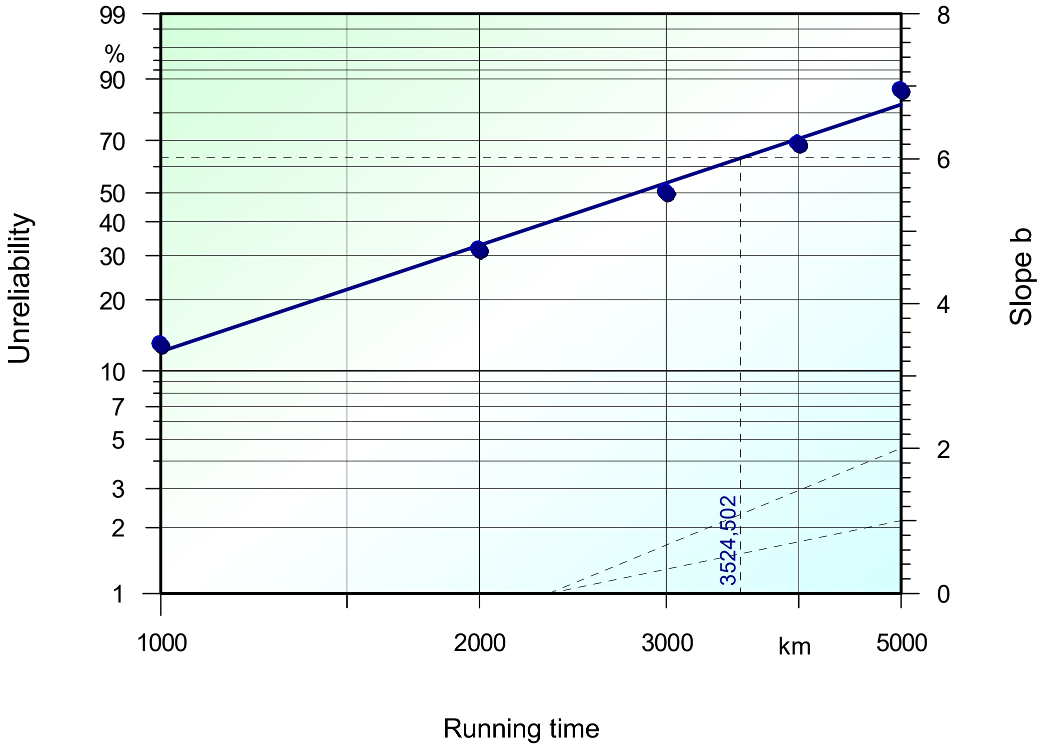
$$H = \frac{G_i}{n + 1} 100\%$$

The exact cumulative frequencies H (also termed median ranks) are determined with the aid of the binomial distribution:

$$0{,}50 = \sum_{k=i}^{n} \frac{n!}{k!(n-k)!} H^k \, (1-H)^{n-k}$$

This equation, however, cannot be transposed to equal H and must therefore be solved iteratively. Nonetheless, the above approximation formula is completely adequate for practical applications.

Once H (for the Y-axis) has been determined for each value, it is possible to draw the Weibull plot with the failure distances (in this case, 1000, 2000, 3000, 4000 and 5000 km):



Step 2: The Weibull function and determining the parameters

In the classic interpretation, Weibull parameters are derived by calculating the best-fitting straight line on the linearised Weibull probability graph /1/.
The points for the best-fitting straight line are determined by transposition of the 2-parameter Weibull function:

$$X = \ln(t)$$

$$Y = \ln\left(\ln\left(\frac{1}{1-H}\right)\right)$$

A best-fitting straight line is generally described by:

$$Y = b\,X + a$$

Referred to our linearisation this corresponds to:

$$Y = b\,X - b\ln(T)$$

b therefore represents both the slope of the best-fitting straight line as well as the shape parameter in the Weibull plot. b and a are generally determined using the known method of the smallest error squares and the above values X and Y. T is then derived from the point of intersection of the best-fitting straight line through the Y-axis, where:

$$a = -b \cdot ln(T)$$

and resolved to

$$T = e^{-\frac{a}{b}}$$

In the literature it is often recommended to perform the linear regression through X and Y instead of from Y and X. The frequency calculation is less susceptible to errors than the running time data. It therefore makes sense to minimise the error squares in X-direction and not in Y-direction (least square method). The formulation is then:

$$Y = \frac{1}{b} X + \frac{1}{b} \ln(T)$$

In practical applications, the differences are negligible referred to b.

In practical applications, b and T are often calculated using **Gumbel** method /4/ where the points on the Weibull plot are weighted differently:

$$b = \frac{0.557}{s_{\log}}$$

$$T = 10^{\left(\sum_{i=1}^{n} \log(t_i)\right)/n + 0,2507/b}$$

were
s_{log} = logarithmic standard deviation

The Gumbel method produces greater values for b than those derived using the standard method. This should be taken into account when interpreting the results.
A further method of determining b and T is the **Maximum Likelihood estimation** (maximum probability) /5/, resulting in the following relationship for Weibull analysis:

$$\frac{\sum_{i=1}^{n} t_i^b \ln(t_i)}{\sum_{i=1}^{n} t_i^b} - \frac{1}{n}\sum_{i=1}^{n} \ln(t_i) - \frac{1}{b} = 0$$

It is assumed that all fault cases correspond to the failure criterion in question. This relationship must be resolved iteratively in order to ascertain b. T can be calculated directly once b has been determined:

$$T = \left(\left(\sum_{i=1}^{n} t_i^{\,b} \right) \frac{1}{n} \right)^{\frac{1}{b}}$$

A further important method is the so-called **Moment Method** and in particular the vertical moment method. In the corresponding deduction from Weibull, published in /11/, the parameters T and b are determined by:

$$b = \frac{\ln(2)}{\ln(\overline{V}_1) - \ln(\overline{V}_2)} \qquad T = \frac{\overline{V}_1}{(1/b)!}$$

were

$$\overline{V}_1 = \frac{1}{2}\left(\frac{1}{n+1} t_m + \frac{2}{n+1}\sum_{i=1}^{n} t_i\right)$$

$$\overline{V}_2 = \frac{1}{2}\left(\frac{1}{(n+1)^2} t_m + \frac{4}{n+1}\sum_{i=1}^{n} t_i - \frac{4}{(n+1)^2}\sum_{i=1}^{n}\left(i\ t_i\right)\right)$$

$$t_m = \sum_{i=1}^{n}(t_i - t_{i-1})$$

This method has the advantage that the computational intricacy and time are relatively low and need not be resolved iteratively as the maximum likelihood method.
By way of example, the corresponding parameters are to be compared for the values 1000, 2000, 3000, 4000 and 5000.

Method	b	T
Best-fitting straight line	1,624	3524
Gumbel	2,018	3468
Maximum likelihood	2,294	3394
Moment method (vertical)	1,871	3281

In this case, the maximum likelihood method produces the steepest slope while the best-fitting line method results in the shallowest gradient. In the following analysis methods considerations are conducted based on the best-fitting straight line, considered to be the standard.

Step 3: Interpretation of results

In view of the often very pronounced dispersion or scatter of the life characteristic, it soon becomes apparent that there is little point in specifying the means of the "running time“. An adequate deduction regarding the failure characteristics of the component in question can be achieved only with the Weibull evaluation. Instead of the mean, the characteristic life T, at which 63.2% of the components fail, is specified. It is optionally indicated with the corresponding perpendicular in the graph.

A further important variable is the shape parameter b which is nothing else than the slope of the straight line in the Weibull net. General one can say:

b < 1 **Early-type failures (premature failures**,)
e.g. due to production/ assembly faults

b = 1 **Chance-type failures (random failures)**
there is a constant failure rate and there is no connection to the actual life characteristic (stochastic fault), e.g., electronic components

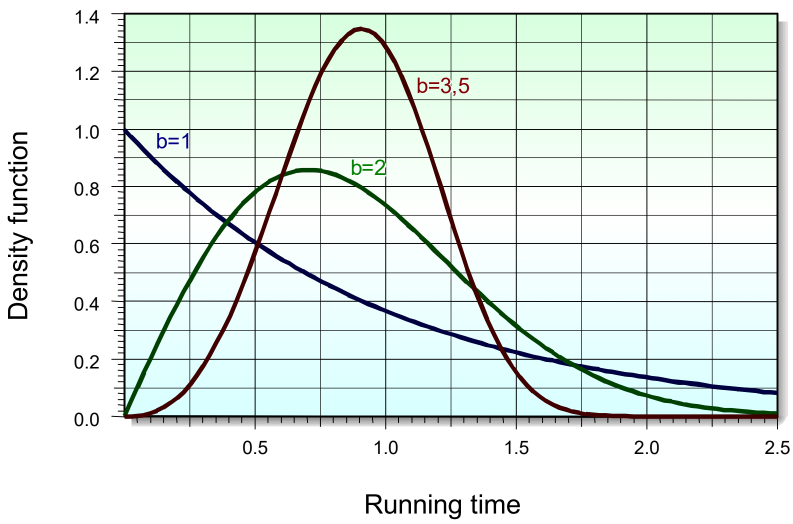
b > 1.. 4 **Time depending (aging effect)**
failures within the design period,
e.g. ball bearings b ≈ 2, roller bearings b ≈ 1.5
corrosion, erosion b ≈ 3 – 4, rubber belt b ≈ 2.5
b > 4 : sometimes called belated failures e.g. stress corrosion, brittle materials such as ceramics, certain types of erosion

The following steps represent special cases:

b = 1 Corresponds to an **Exponential distribution** $H = 1 - e^{-\lambda t}$
Constant failure rate

b = 2 Corresponds to **Rayleigh distribution**, linear increase in failure rate

b = 3.2..3.6 Corresponds to **Normal distribution**



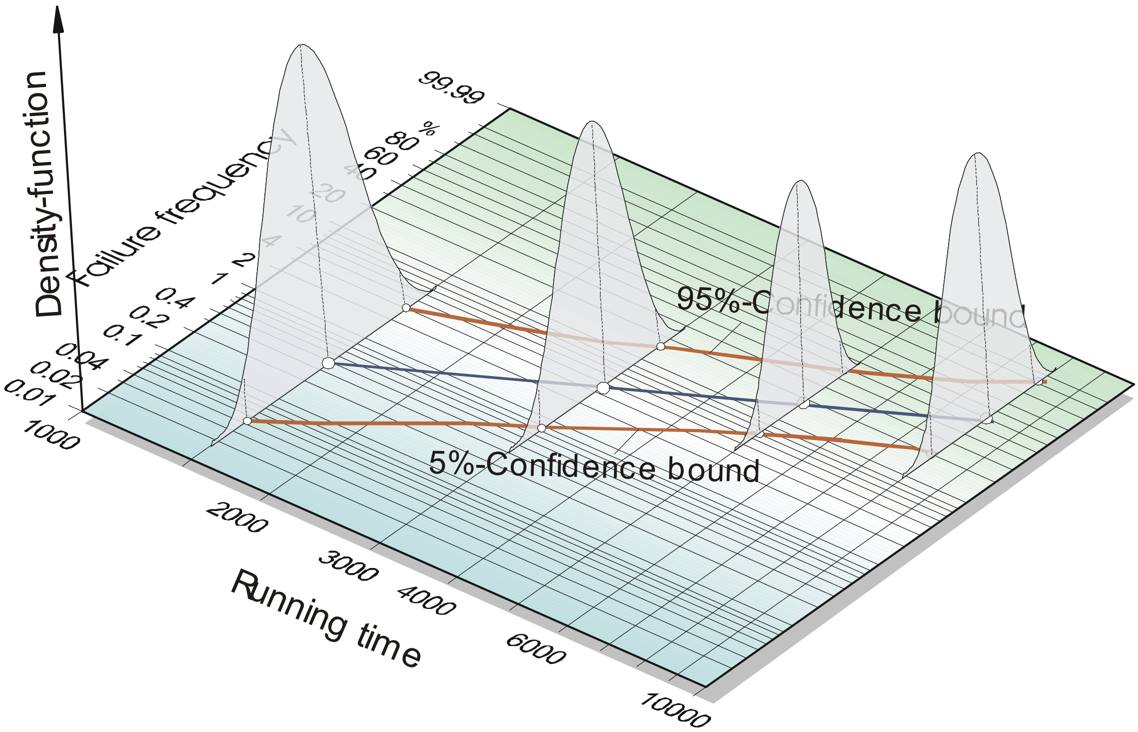
Here, the term early failure (b<1) is ambiguous as it is also possible that a wear characteristic only becomes apparent through a production fault (e.g. excessively high roughness values of a bearing assembly). Despite b>2, there may also be "premature failures" in this case as the damage already occurs after a very short period of time. Wherever possible, the terms and definitions should therefore be used in context. It is essentially possible to state that there is no dependency on running time at values of $b \leq 1$.

In practical applications it is often the case that the failure data is based on a mixed distribution. This means that, after a certain "running time“, there is a pronounced change in the fault increase rate due to the fact that the service life is subject to different influencing variables after a defined period of time. The various sections should therefore be observed separately and it may prove advantageous to connect the individual points (failures) on the graph instead of one complete best-fitting straight line (refer to section entitled *Mixed distribution*).

It is desirable to have available a prediction of the reliability of a component before it goes into series production. Fatigue endurance tests and simulation tests are conducted under laboratory conditions in order to obtain this prediction. For time reasons these components are subjected to increased stress load (often also due to safety factor reasons e.g. factor 2..3) in order to achieve a time-laps effect. Entering these service life characteristics in the Weibull plot will result in a shift in the best-fitting straight line to the left compared to a test line, for which a normal stress load was used. This is to be expected as components subjected to higher loads will fail earlier. However, if the progression of these best-fitting straight lines is not parallel, but increase at a different rate, this indicates a variety of failure mechanisms in relation to the test and real applications. The test is therefore not suitable.

Confidence bounds

The Weibull evaluation is based on what may be viewed as a random sample. This in turn means that the straight line on the Weibull plot only represents the random sample. The more parts are tested or evaluated, the more the "points" will scatter or disperse about the Weibull straight line. It is possible to make a statistical estimation as to the range of the populations. A so-called "confidence bound" is introduced for this purpose. These bounds are generally defined through the confidence level, mostly at P_A=90%. This means the upper confidence limit is at 95% and the lower at 5%. The following example shows the two limit or bound lines within which 90% of the population is located.



The confidence bound is calculated based on the binomial distribution as already discussed (chapter *Determining failure frequencies*), however instead of P_A=0.50, in

$$0{,}05 = \sum_{k=i}^{n} \frac{n!}{k!(n-k)!} H^k (1-H)^{n-k} \qquad 0{,}95 = \sum_{k=i}^{n} \frac{n!}{k!(n-k)!} H^k (1-H)^{n-k}$$

this case P_A=0.05 is used for the 5% confidence bound and P_A=0.95 for the 95% confidence bound:

H represents the required values of the confidence bound. However, the problem in this

case is also that this formula cannot be resolved analytically for H.
It is also possible to calculate the confidence bound using the Beta distribution with the corresponding density function, represented vertically, using the rank numbers as parameters. More commonly, tabular values are available for the F-distribution. By way of transformation, the confidence bound can be determined using this distribution.

$$V_{i,top} = 1 - \frac{1}{1 + \frac{i}{n-i+1} F_{2i,2(n-i+1),\frac{1-\alpha}{2}}} \qquad V_{i,bottom} = \frac{1}{\frac{n-i+1}{i} F_{2(n-i+1),2i,\frac{1-\alpha}{2}} + 1}$$

The progression of the confidence limits moves more or less apart in the lower and upper range. This indicates that the deductions relating to the failure points in these ranges are less accurate than in the mid-upper section.
In the same way as the best-fitting straight line, the confidence bound must not be extended substantially beyond the points.
Reference is made in particular to /1/ for more detailed descriptions.

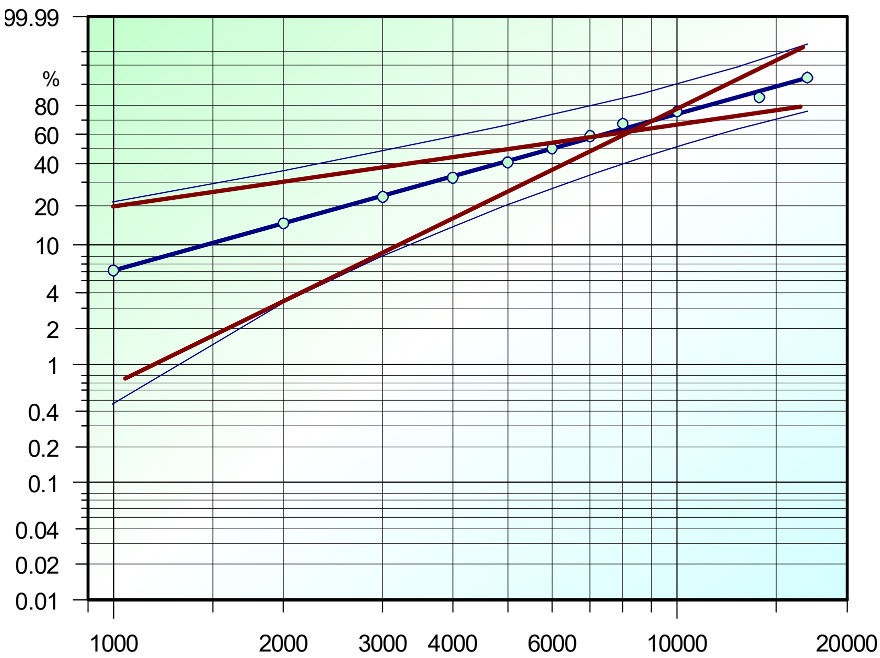
The confidence bound of the slope b

There is an approximation formula especially for the confidence interval of the parameter b:

$$b \frac{1}{1+\sqrt{\frac{1,4}{n}}} \le b \le b\left(1+\sqrt{\frac{1,4}{n}}\right)$$

Relation for other confi-dence levels

90%	$1+\sqrt{1,4/n}$
95%	$1+\sqrt{2,0/n}$
99%	$1+\sqrt{3,4/n}$



The confidence bound of b is used, among other things, to decide on a mixed distribution.

The confidence bound of the characteristic life

The following relationship with using the χ^2-distribution is recommended for the characteristic lifetime T:

Excel-Formula for χ^2-distribution
=CHIQU.INV(0,05; 2*n) and
=CHIQU.INV(0,95; 2*n)

$$T\left(\frac{2n}{\chi^2_{2n,1-\alpha/2}}\right)^{1/b} \le T \le T\left(\frac{2n}{\chi^2_{2n,\alpha/2}}\right)^{1/b}$$

Confidence-level 90% ⇨ α = 10%

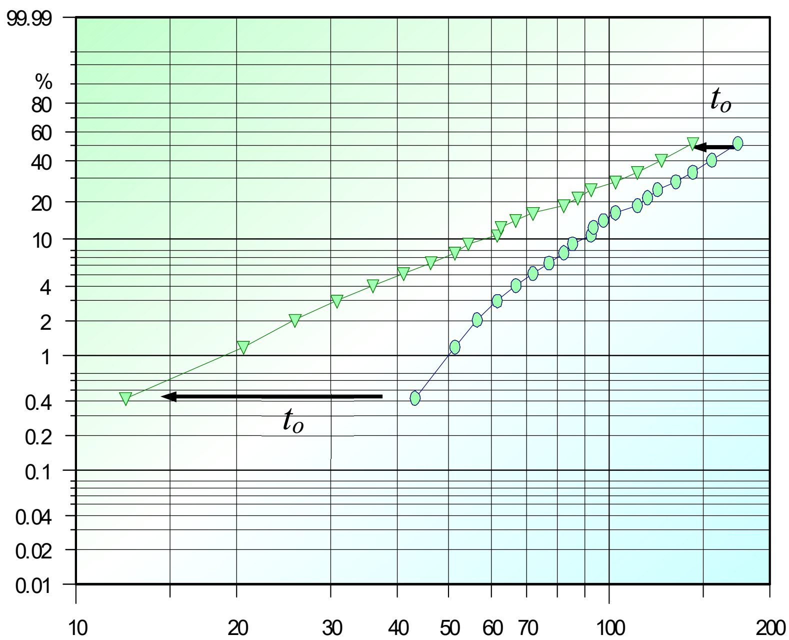
Also, here this confidence bounds are used, among other things, to decide on a mixed distribution.

T can be exchanged with t_{10} to get the limit for an t_{10} requirement.

The 3-parametric Weibull distribution with t_o

A component requires a certain period of time before wear occurs, for example brake pad
wear. This time is called a failure-free time.

There is no distinct mathematical formula for this purpose. A fundamental requirement in connection with the failure-free period t_o is that it must lie between 0 and the value of the first failed part. t_o usually occurs very close before the value of the first failure. The following method suggests itself: t_o p asses through the intervals Interval between t>0 and the first failure t_{min} in small steps and the correlation coefficient of the best-fitting straight line. is calculated at each step. The better the value of the coefficient of correlation, the more exact the points lie on a straight line in the Weibull plot. t_o is then the point at which the value is at its highest and therefore permits a good approximation with the best-fitting line.



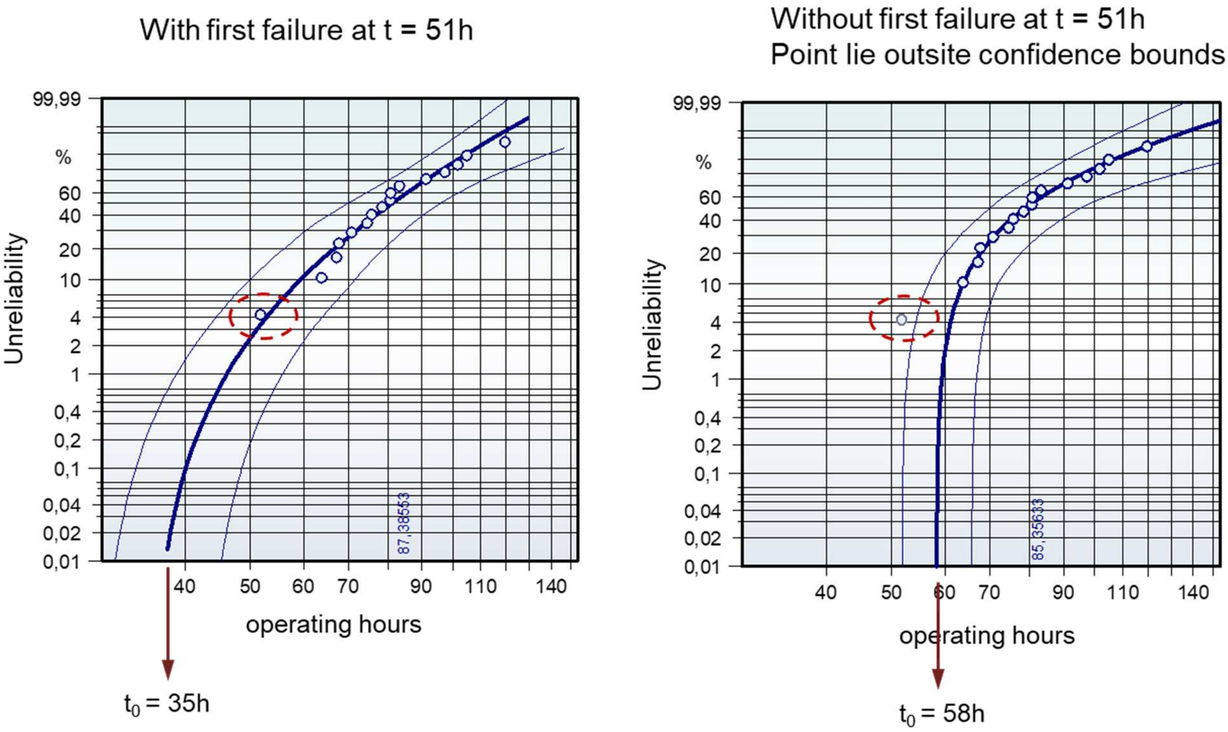
In graphic terms, this means nothing else than that the points in the Weibull plot are applied shifted to the left by the amount t_o, see graph in the right:

The points then result in the best linearity. This is due to the fact that due to the logarithmic X-axis, the front section is stretched longer than the rear section, thus cancelling out the curvature of the points to the right.

It is, of course, possible to test statistically the coefficient of correlation of the best-fitting straight line with t_o using the relevant methods /2/, establishing whether the failure-free time is applicable or not (F-test for testing the linearity or t-test for the comparison of the regressions with and without t_o). In view of the numerous possible causes of the curved line progression, a statistically exact hypothesis test is not

worthwhile in the majority of practical applications. However, the method just described should be used to check whether the correlation coefficient of the best-fitting straight line with t_o lies at $r \geq 0.95$.
A mixed distribution is very probably if, instead of the slightly rounded off progression as previously described, a significant kink to the right can be seen at only one point with otherwise different straight line progressions. The same applies particularly to a kink to the left, in connection with which there is no possibility of a failure-free time t_o (with the exception of defective parts with negative t_o). Please refer to the section entitled *Mixed distribution.*
Note:
The shift of the points by t_o is not represented in the Weibull plot but rather implemented only by way of calculation.

Example of wear-affected component where to is very much depending on the 1st point. This is often an outlier, e.g. a process- or manufacturing problem!:



Other non-linear Weibull-Functions

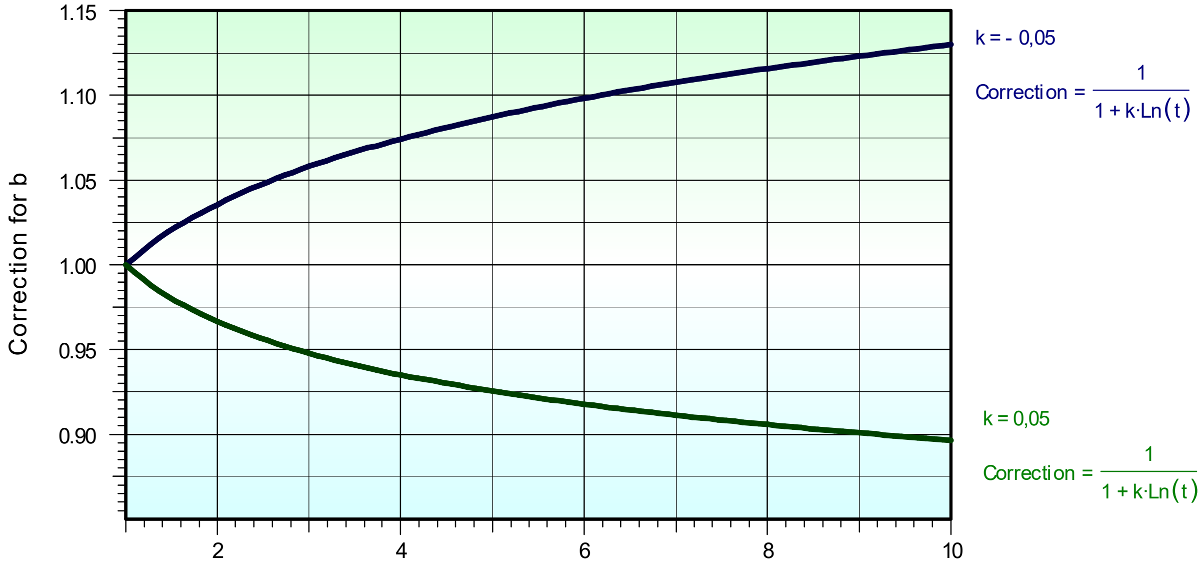
Often there are non linear Weibull-distributions, which can not be satisfying described with the 3-parametric function with to. In particular the course for a very long-life span flattens steadily. This is the case if the failure-probability decreases by other connections than the normal failure cause (like fatigue, aging etc.). The reason is the often the death rate because of accidents. On this consideration, the bend is relatively steady in the Weibull diagramme for the time, almost constant. With the standard 3-parametrig Weibull function with to at the beginning the bend is high and later runs out. A function or an extension of the Weibull function with the following attributes is searched:

- Curve progression with very steady bend.
- Representation possible convex or concave

These requirements can be realised with the following term in the exponent from b:

$$\frac{1}{1+k\ \ln(t)}$$

The parameter k shows the strength of the bend. If this is positive, a decreasing gradient b arises. If it is negative, there originates an increasing gradient. The following example shows for k =-0,05 and k=0,05 the courses:



At the start with t=1 is the correction=1. The gradient is here the original one. The interpretation with regard to b refers to the beginning, while ascertained b is to be interpreted for 3-parametrige Weibull function approximately on the right outlet of the curve. The Weibull function with correction factor dependents on time, b becomes therefore to:

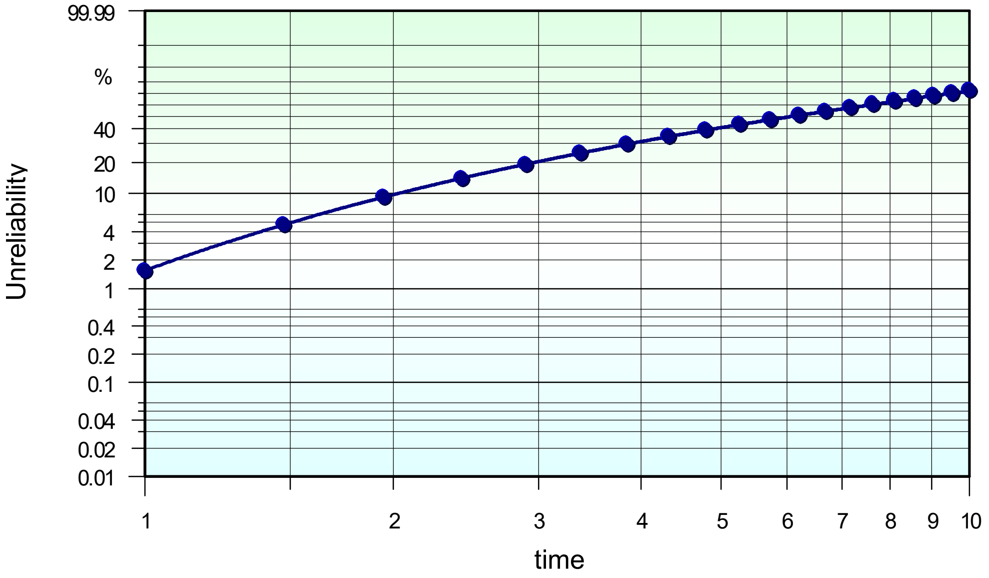
$$H = 1 - e^{-\left(\frac{t}{T}\right)^{\frac{b}{1+k\ \ln(t)}}}$$

The logarithm ensures that at high running time the correction grows not excessively. With concave course with negative k the denominator $1+k\ ln(t)$ can not be less or equal 0. In addition, it can happen that this enlarged Weibull function goes more than 100%. Both show an inadmissible range. This extension (correction) is not based on derivation of certain circumstances, like the death rate. Hereby merely one function should be made available for concave or convex curve courses with which one can better describe the course of a non-linear Weibull curve. The measure of the goodness of this function is the correlation coefficient r. The higher this is, one can use better this new Weibull function also for extrapolating at higher times than data points exist. Example for a degressive Weibull - curve:

$T = 8{,}0746 \quad b = 1{,}99 \quad k = 0{,}308$

$$H = 100\% \cdot \left(1 - e^{-\left(\frac{t}{T}\right)^{\frac{b}{1 + k \cdot Ln(t)}}}\right)$$

$r = 1$



The parameter k must be determined iteratively. As the first estimate for the start of the iteration can be determined b from straight regression. Also, the characteristic lifetime T.

Another beginning is the use of an exponential function for a non-linear curve.

$$Y' = \alpha \cdot e^{\varphi X}$$

With this beginning can be illustrated non-linear courses, in particular roughly steadily stooped, ideally. However, this function is bent first on the left instead of on the right. Therefore, the transformation occurs more favourably points with:

$$Y' = -\ln\left(\ln\left(\frac{1}{1-H}\right)\right) + \tau$$

(see chapter Determination of the Weibull parameters)

With the Offset $\tau = Y[n_a]+1$ for the last point of failure. Herewith one reaches that the points are reflected round the X axis and the function is bent on the right. If one uses now X and Y' in the exponential function, thus originates, in the end

$$H = 1 - e^{-e^{-\left(\alpha\, e^{\varphi \ln(t)} - \tau\right)}}$$

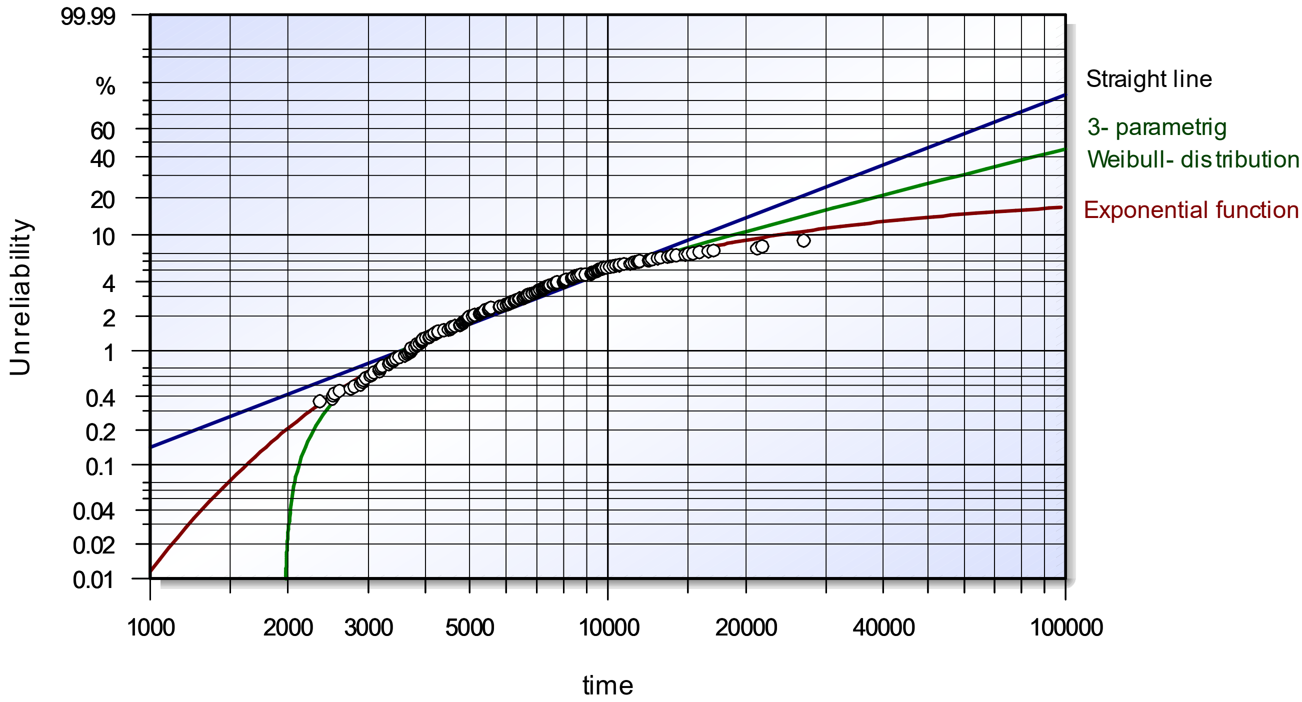
If one combines the exponent to x, this new equation relates to the in professional circles well known extreme value distribution type 1 from Gumbel:

$$H = 1 - e^{-e^{-x}}$$

The suitable inverse function of the new exponential form is:

$$t = e^{\frac{1}{\varphi}\ln\left(\frac{\tau - \ln(\ln(1/(1-H)))}{\alpha}\right)}$$

With this equation, it is possible for example, to calculate the t_{10}-value (B10).
The following example shows the differences compared with a concrete failure behaviour:



The classical straight line shows the worst approxiamtion of the failure points, in particular up to 10,000 km. The 3-parametric Weibull distribution with to is better quite clearly, however, shows in the outlet of the last failure points still too big divergences. A statement about the failure likelyhood, e.g., with 100,000 km would deliver too high values. In this case approx. 45% of failures should be expected. However, these have not appeared later.
Only the exponential beginning was satisfactory. The result became even better if one used only the rear upper points (approx. 2/3 of the whole number) for the fitting of the Weibull function. Premature failures in the quite front area have been already taken out of the representation (process failures).
The Weibull-exponential function shows no typical down-time to T or gradient b it would be to be interpreted. α, φ and τ are suitable only to form and situation parametre of this function. An enlargement from α shifts the curve to the right. This is comparable with the behaviour if T is increased in 2-parametrigen Weibull function. Indeed, also shift φ and τ the curve. An enlargement of the respective values proves here a link movement, and the course is bent, in addition, more precipitously and stronger.
Which approach has to be selected finally? The adaptation of the generated curve to the failure points is judged at first optically. If the steadily stooped course fits to the points, one decides with the help of the correlation coefficient from the method of the

least square fit between 3-parametrig Weibull distribution or the exponential function. The closer the correlation coefficient lies to 1, the better the function is suitable. For the adaptation of the function to concrete failure points it may be suitable to let out early failures or extreme points.

Both shown approaches are recommended if the failure points in the middle range are steadily and convex. If there are mixed distributions the method of splitting in several divisions is recommended.

Other characteristic variables

Failure rate

The failure rate indicates the relative amount of units that fail in the next interval (t+dt). It is a relative parameter and should not be confused with the absolute failure quota. Since the remaining number of decreases in time the absolute failures will also decrease at a constant failure rate.

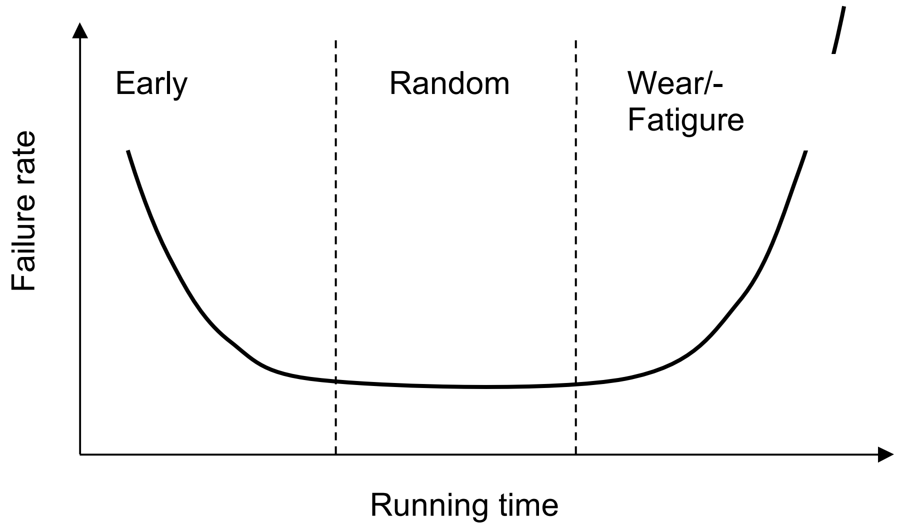
$$\lambda_T = \frac{b}{T}\left(\frac{t}{T}\right)^{b-1} \qquad \lambda_T = \frac{b}{T-t_o}\left(\frac{t-t_o}{T-t_o}\right)^{b-1}$$

2-parameter 3-parameter

This result in a constant failure rate for b=1 that is dependent only on T:

$$\lambda_{T,b=1} = \frac{1}{T}$$

A constant failure rate is often encountered in the electrical industry. The graphic representation of different failure rates is often referred to as a bathtub life curve:



Each of the three ranges are based on different causes of failure. Correspondingly different measures are necessary to improve the reliability.
Further details can be found in the section *Interpretation of results*.

Expected value (mean)

The mean t_m of the Weibull distribution is rarely used.

$$t_m = T\,\Gamma\left(1+\frac{1}{b}\right) \qquad t_m = (T-t_o)\,\Gamma\left(1+\frac{1}{b}\right)+t_o$$

2-parameter 3-parameter

In the literature the expected value is described as

- *MTTF* (Mean Time To Failure) for non-repaired units
- *MTBF* (Mean operating Time Between Failures) for repaired units

For a constant failure rate with b=1, the expectancy value *MTTF* is derived from the reciprocal of the parameter λ (*MTTF*=1/λ) . This is **not** generally valid for the Weibull distribution.

Standard deviation

The standard deviation of the Weibull distribution is also obtained with the aid of the gamma function:

$$\sigma = T\sqrt{\Gamma\left(1+\frac{2}{b}\right)-\Gamma\left(1+\frac{1}{b}\right)^2} \qquad \sigma = (T-t_o)\sqrt{\Gamma\left(1+\frac{2}{b}\right)-\Gamma\left(1+\frac{1}{b}\right)^2}$$

2-parameter 3-parameter

Variance

Corresponding to the standard deviation the formula is:

$$\sigma^2 = T^2\left(\Gamma\left(1+\frac{2}{b}\right)-\Gamma\left(1+\frac{1}{b}\right)^2\right) \qquad \sigma^2 = (T-t_o)^2\left(\Gamma\left(1+\frac{2}{b}\right)-\Gamma\left(1+\frac{1}{b}\right)^2\right)$$

2-parameter 3-parameter

Availability

The availability or permanent availability is the probability of a component to be in an operable state at a given point in time t. The permanent availability of a component is determined by:

$$A_D = \frac{MTTF}{MTTF + MTTR}$$

with the expectancy value $MTTF$ already defined and the mean failure or repair time $MTTR$ (Mean Time to Repair). The unit of $MTTR$ must be the same as for $MTTF$. If, for example, the $MTTF$ is defined in kilometres, the time specification for $MTTR$ must be converted to the equivalent in kilometres. It is necessary to define an average speed for this purpose.

It is possible to determine the system availability using Boolean operations (please refer to the section headed *Overall availability of systems*).

t_{10}-Lifetime

The lifetime, up to which 10% of the units fail (or 90% of the units survive); this lifetime is referred to in the literature as the reliable or nominal life B_{10}.

$$t_{10} = T\left(\ln\left(\frac{1}{1-0{,}1}\right)\right)^{\frac{1}{b}} = 0{,}1054^{\frac{1}{b}} \cdot T \quad \text{where } t_o \quad t_{10} = (T-t_o)\cdot 0{,}1054^{\frac{1}{b}} + t_o$$

t_{50}-Lifetime, median

$$t_{50} = T\left(\ln\left(\frac{1}{1-0{,}5}\right)\right)^{\frac{1}{b}} = 0{,}6931^{\frac{1}{b}} \cdot T \quad \text{where } t_o \qquad t_{10} = \left(T - t_o\right) \cdot 0{,}6931^{\frac{1}{b}} + t_o$$

t_{90} – Lifetime

$$t_{90} = T\left(\ln\left(\frac{1}{1-0{,}9}\right)\right)^{\frac{1}{b}} = 2{,}303^{\frac{1}{b}} \cdot T \quad \text{where } t_o \qquad t_{10} = \left(T - t_o\right) \cdot 2{,}303^{\frac{1}{b}} + t_o$$

4. Comparison of 2 distributions

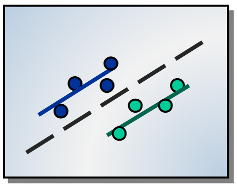
The question discussed here is: Is one design, system or component more reliable than another. For example, has the introduction of an improvement measure verifiably extended the service life. The differences can be quantified by the characteristic life T -> T_1 / T_2.

Statistical tests define a so-called null hypothesis:
⇨ Null hypothesis: The distributions are identical.
⇨ Alternative hypothesis: The distributions are different.

Note: Although the question is usually to look for a difference, one must define the null hypothesis and the goal is to reject it. The question is whether differences are significant or cased by random. To answer the hypotheses, the following method can be used:

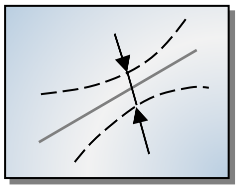
Step 1

Determination of mean parameters



$$T_m = e^{\frac{\ln(T_1) + \ln(T_2)}{2}} \qquad b_m = (b_1 + b_2)/2$$

Step 2



Define the confidence bounds
Standard = 90% ⇨ α = 10%

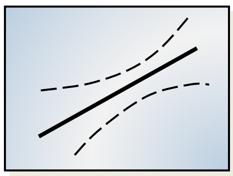
$\alpha/2$ = 5 %; $1 - \alpha/2$ = 95 %

Step 3

Determine the confidence bounds of the Weibull parameters.
Form parameter b

$$b_m \frac{1}{1+\sqrt{\frac{1,4}{n}}} \leq b_m \leq b_m \left(1+\sqrt{\frac{1,4}{n}}\right)$$

n = number of faults



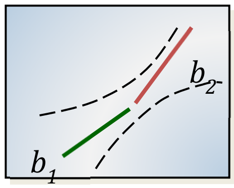
Characteristic lifetime T combined straight line

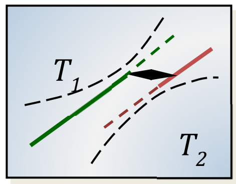
$$T_m \left(\frac{2n}{\chi^2_{2n,\alpha/2}}\right)^{1/b} \leq T_m \leq T_m \left(\frac{2n}{\chi^2_{2n,1-\alpha/2}}\right)^{1/b}$$

Step 4

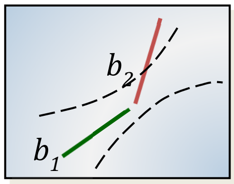
Lie b_1 and b_2 respective T_1 and T_2 inside the confidence bounds of the parameters?

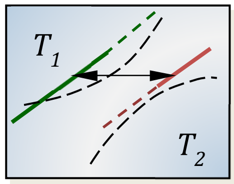
Yes : ⇨ The null hypothesis, that the distributions are equal can not be rejected





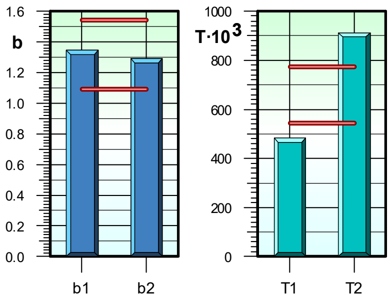
No : ⇨ The null hypothesis has to be rejected ⇨ the distributions





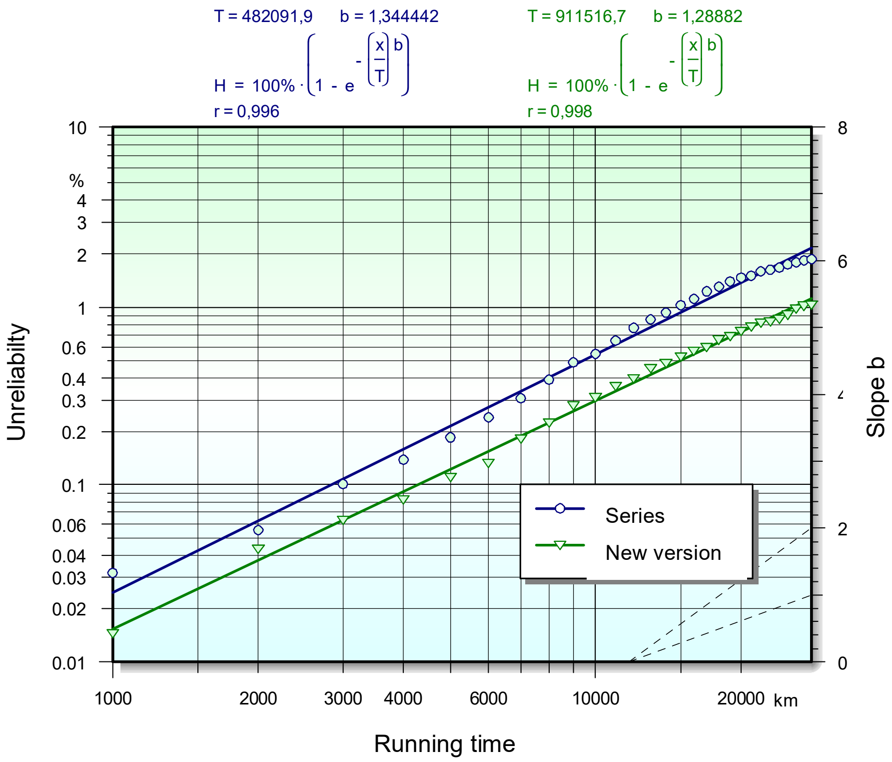
Hint: It is sufficient that either b *or* T *are outside the confidence bounds*

Example of a field warranty problem:



b ges	1,32
b1	1,34
b2	1,29
bmin (90%)	1,09
bmax (90%)	1,54
T ges	658290,31
T1	481974,38
T2	911284,95
Tmin (90%)	542668,20
Tmax (90%)	773912,42

Different distributions Yes



In this example, the individual characteristic lifetimes overshoot the confidence bound of the overall straight line.

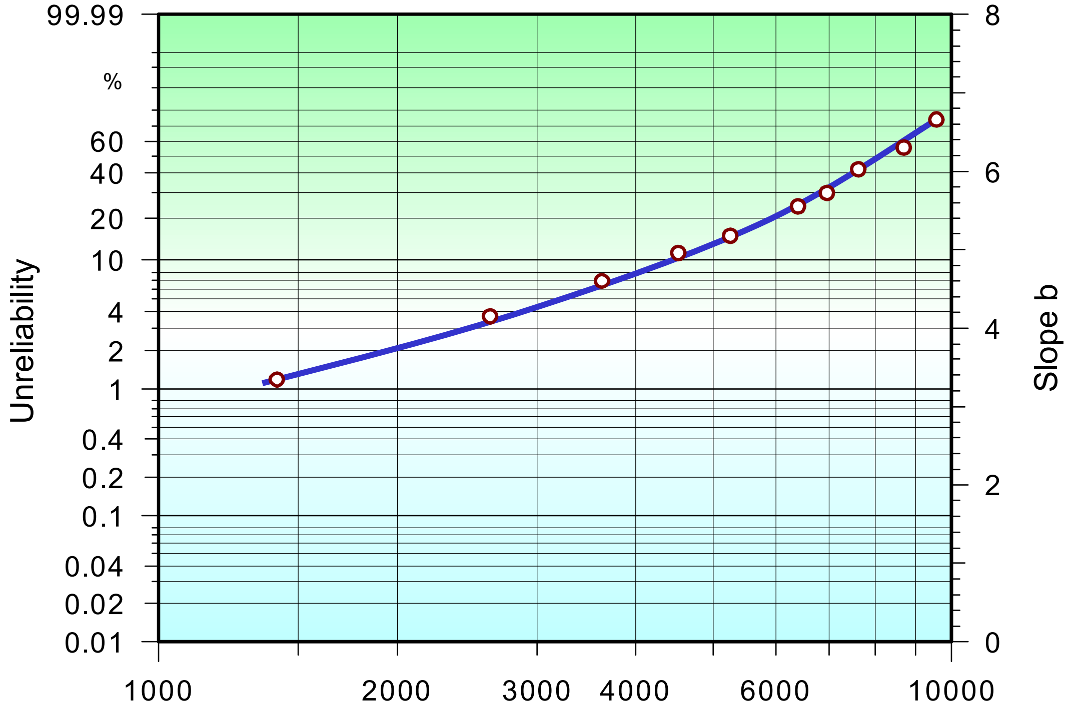
5. Mixed distribution

In connection with different failure mechanisms it is possible that the progression of the Weibull plot is not a straight line but rather a curve. If it is possible to attribute the reason for the component failure to different reasons, the corresponding "fault groups" are evaluated separately in the Weibull plot. The following formula applies to two different Weibull distributions:

$$H = \frac{n_1}{n} H_1 + \frac{n_2}{n} H_2 = \frac{n_1}{n}\left(1 - e^{-\left(\frac{t}{T_1}\right)^{b_1}}\right) + \frac{n_2}{n}\left(1 - e^{-\left(\frac{t}{T_2}\right)^{b_2}}\right)$$

If evaluation of the component failures is not possible, distinct confirmation of a mixed distribution may be determined only with difficulty as the individual points are always scattered or dispersed about the best-fitting straight line.

As in virtually all statistical tests, the question arises as to whether the deviations of the failure points are coincidental or systematic. The following procedure is used: 2 best-fitting lines are determined from the points in the initial section and from the points in the rear section, starting with the first 3 points, i.e. if an evaluation has 10 failures, the rear section contains the last 7 points.



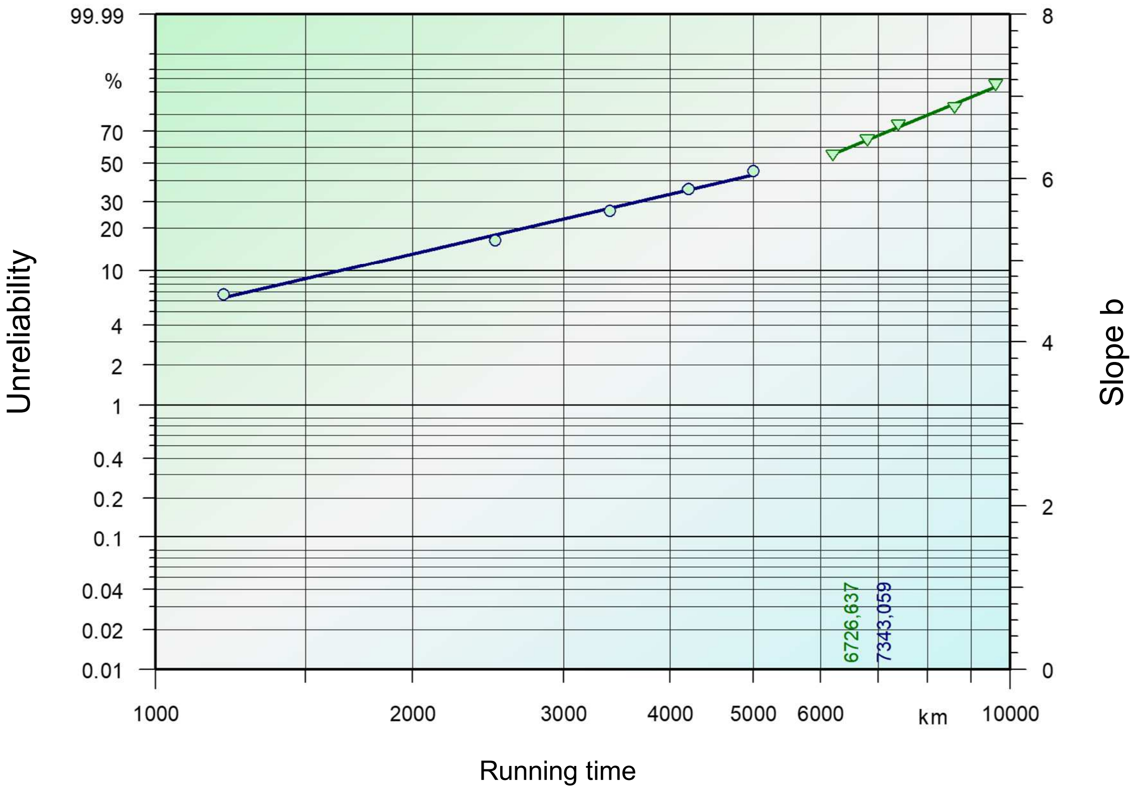
In the next step, the first 4 points and the last 6 points are combined and so on. This procedure is continued until the second section contains only 3 points. The correlation coefficients of the respective sections are then compared and the possible "separating point" determined, at which the correlation coefficient of both best-fitting straight lines are the best. All that is necessary now is to perform a test to determine whether both sections represent the progression of the failures better than a best-fitting straight line over all points together. Different options are available for this purpose. In the same way as the comparison of two distributions, it is useful to examine the two slopes up to the confidence bound of the slope of the overall straight line. Please refer to section "The confidence bound of the slope b".

$$b_m \frac{1}{1+\sqrt{\frac{1,4}{n}}} \leq b_m \leq b_m \left(1+\sqrt{\frac{1,4}{n}}\right)$$

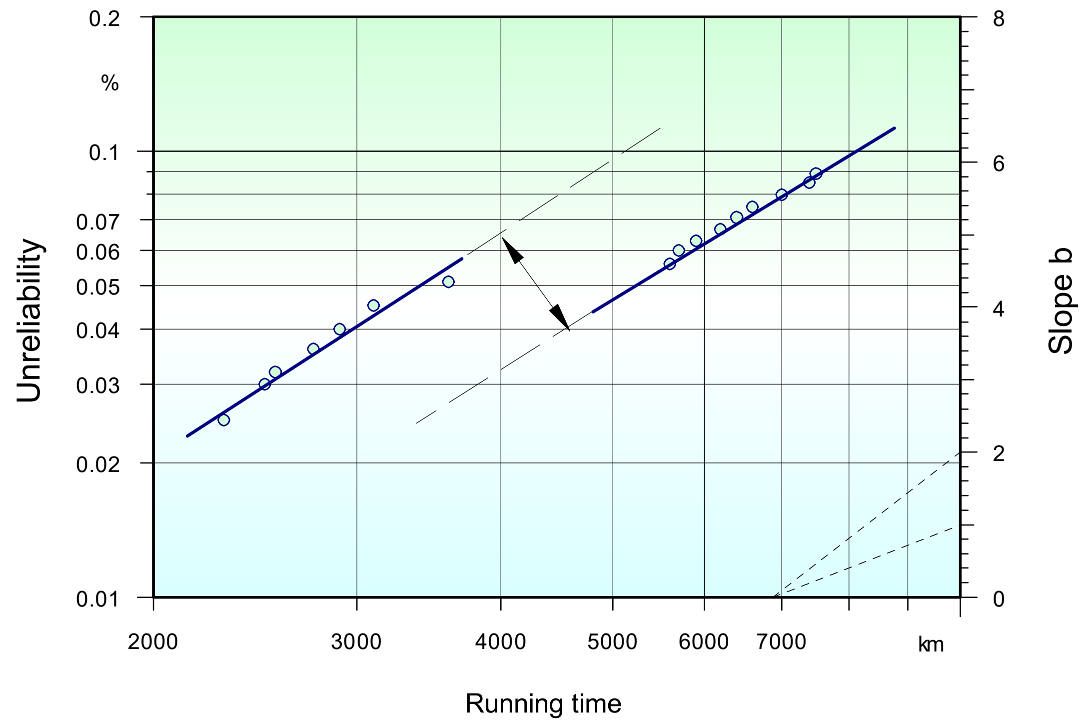
The hypothesis that there is a mixed distribution can be confirmed if the flatter slope of both subsections lies below the lower confidence bound and/or the steeper slope above the upper confidence bound.

Example

Although, strictly speaking, the procedures described above apply as from a random sample quantity of n≥50, to simplify the explanation, the following example deals with 10 failures. The following operational distances (mileage) were recorded up to the point of failure: 1200, 2500, 3400, 4200, 5000, 6200, 6800, 7400, 8600, 9600 km. The method just described produces a separation of both sections between points 5 and 6, see diagram. The slope of the first section is calculated at 1.5 while that of the second section is 2.7. In accordance with the relationship presented at the beginning, the confidence bound of the slope for the entire range at α□= 90% extends over $1.03 < b_{total} < 2.43$. The slope of section 2 therefore exceeds the confidence bound in the upward range and the hypothesis of a mixed distribution can be confirmed. Nevertheless, the result is tight. If the last failure point were set to 10200 km instead of 9600 km the confidence bound would no longer be exceeded.



There are, however, mixed distributions in which the slopes of both sections lie very close together but offset:



The test used to date would possibly not detect a mixed distribution although the overall slope over all points is not as steep as the individual sections. The concrete example of the above distribution of a starter, however, had definitively different failure causes. In this case, it is advisable to additionally check the respective characteristic life T in terms of the confidence bound of the overall straight line. This can be achieved by the following approximation formula:

$$T_m \left(\frac{2n}{\chi^2_{2n,\alpha/2}} \right)^{1/b} \leq T_m \leq T_m \left(\frac{2n}{\chi^2_{2n,1-\alpha/2}} \right)^{1/b}$$

The hypothesis that a mixed distribution exists applies also in this case when the two T of the subsections transgress the confidence bound of the overall best-fitting straight line at one of the two sides.

The alternative hypothesis that there is no mixed distribution is not permitted with this method as there are also other reasons for a curved progression. For example, it is possible that a progression kinked to the right may represent a failure-free time t_o or the data run out (please refer to chapter "Prognosis"). In addition, it is also not fundamentally possible to assume that the two distributions extend over different distance ranges. "Interlocked" distributions cannot be detected.

6. Test for Weibull distribution

Generally, data can be checked to establish whether they obey a certain type of distribution. In the case of the Weibull distribution, it should be borne in mind that it is a universal distribution that contains other distributions (e.g. lognormal distribution).
As part of the Kolmogorov-Smirnov test, the available data are compared (empirically) to the Weibull distribution function. The parameters b, T and possibly t_o of this function, however, are estimated from the available data (see section entitled "Determining Weibull parameters).
If, instead of the random variables T with the function:

$$H = 1 - e^{-\left(\frac{t - to}{T - to}\right)^b}$$

the random variable:

$$X = \left(\frac{t - t_o}{T - t_o}\right)^b$$

is taken into consideration, the resulting exponential distribution is:

$$H = 1 - e^{-X}$$

Based on this consideration, the Kolmogorov-Smirnov test (KS-Test) can be applied for the exponential distribution with an unknown parameter. The function checks whether thy hypothetical distribution function $H_{(X)}$ corresponds to the actual distribution. The null hypothesis is:

Ho: The distribution is a Weibull distribution

The deviation is compared with respect to the frequencies of the sorted data (ranking i). $H=i/(n+1)$ applies for $n \geq 50$. Since the empirical data represent a "step function", in addition to the position i, the deviation at the previous position $i\text{-}1$ must be checked compared to the function value H_i.
The hypothesis is not rejected if a maximum distance D between the empirical frequency and the Weibull function is not exceeded.

$$D = \max\left[\left|\left(\frac{i - 0{,}3}{n + 0{,}4}\right) - \left(1 - e^{-\left(\frac{t - t_o}{T - t_o}\right)^b}\right)\right|, \left|\left(\frac{i - 1 - 0{,}3}{n + 0{,}4}\right) - \left(1 - e^{-\left(\frac{t - t_o}{T - t_o}\right)^b}\right)\right|\right] \quad \text{for} \quad n < 50$$

$$D = \max\left[\left|\left(\frac{i}{n + 1}\right) - \left(1 - e^{-\left(\frac{t - t_o}{T - t_o}\right)^b}\right)\right|, \left|\left(\frac{i - 1}{n + 1}\right) - \left(1 - e^{-\left(\frac{t - t_o}{T - t_o}\right)^b}\right)\right|\right] \quad \text{for} \quad n \geq 50$$

The critical values for D from conventional tables are suitable only when the parameter of the exponential distribution is known. While using Monte Carlo methods, Lilliefors /16/ determines critical values for the case that the parameter must be estimated from the random sample. The advantage of the KS test is that it can also be applied to small random samples. However, complete samples are assumed.
Example: The following failure data are available where $t_o = 0$. The Weibull parameters were determined using the best-fitting straight-line method and are T=56.45 and b=1.4.

i	t	$\frac{i-0.3}{n+0.4}$	$1-e^{-\left(\frac{t}{T}\right)^b}$	Δ at i	Δ at i-1
1	13	0.10938	0.12050	0.011	0.121
2	24	0.26563	0.26108	0.005	0.152
3	31	0.42188	0.35127	0.071	0.086
4	55	0.57813	0.61885	0.041	0.197
5	78	0.73438	0.79238	0.058	0.214
6	91	0.89063	0.85774	0.033	0.123

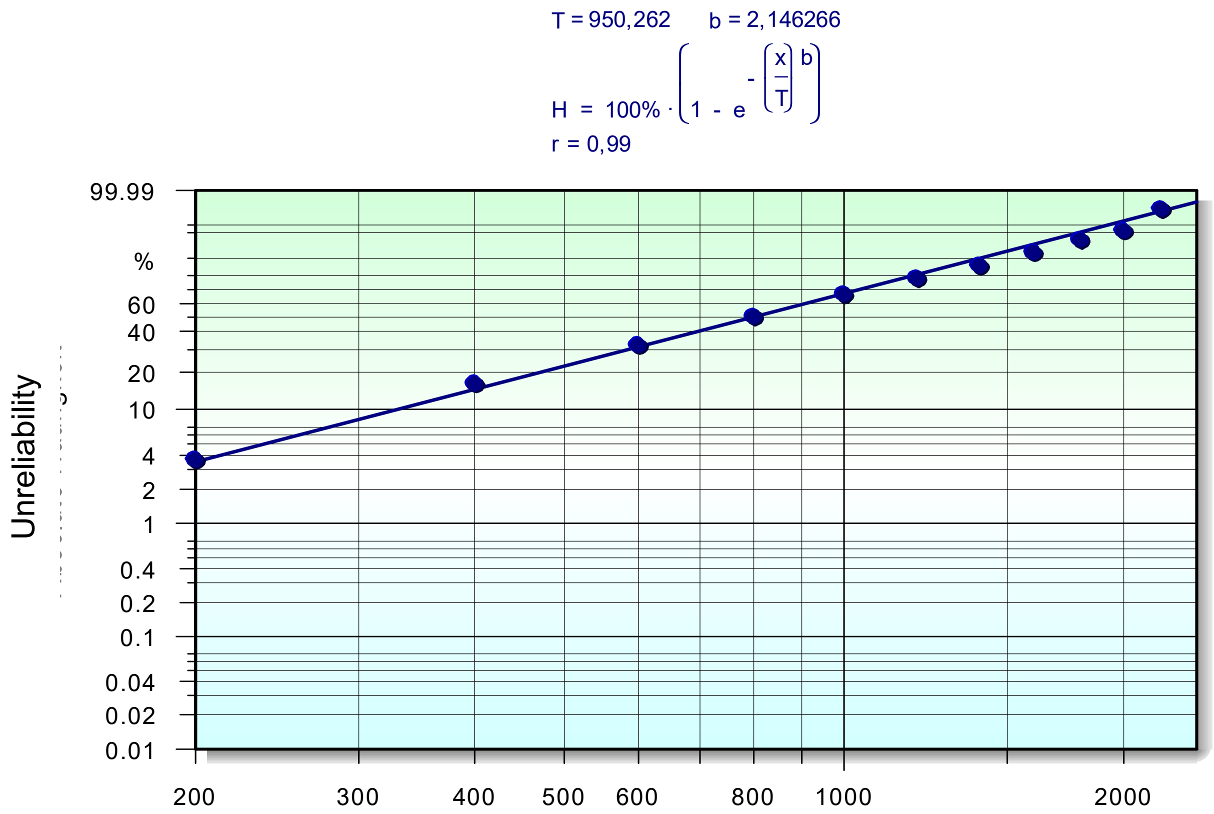
D_{max} is therefore 0.214. According to the table (see annex) the critical value for a significance level of α = 5% is $KS_{\text{crit } 6.5\%}$ = 0.408. The hypothesis that the distribution is a Weibull distribution is confirmed as $D_{max} < KS_{\text{crit } 6.5\%}$.
The possible causes must be determined if there is no Weibull distribution. The representation of the failure frequencies in the Weibull plot is therefore not incorrect, however, it must not be inferred to other failure values with the determined parameters or extrapolated.

7. Monte Carlo Simulation

A data record can be created for certain defined Weibull parameters using the so called Monte Carlo simulation. The frequency H is determined by means of a random generator conforming to the requirement: 0<H<1. However, the maximum frequency can be further restricted by sorting out the parameters outside the required range. The corresponding running times are then calculated with the inverse function:

$$t = T\left(\ln\left(\frac{1}{1-H}\right)\right)^{\frac{1}{b}}$$

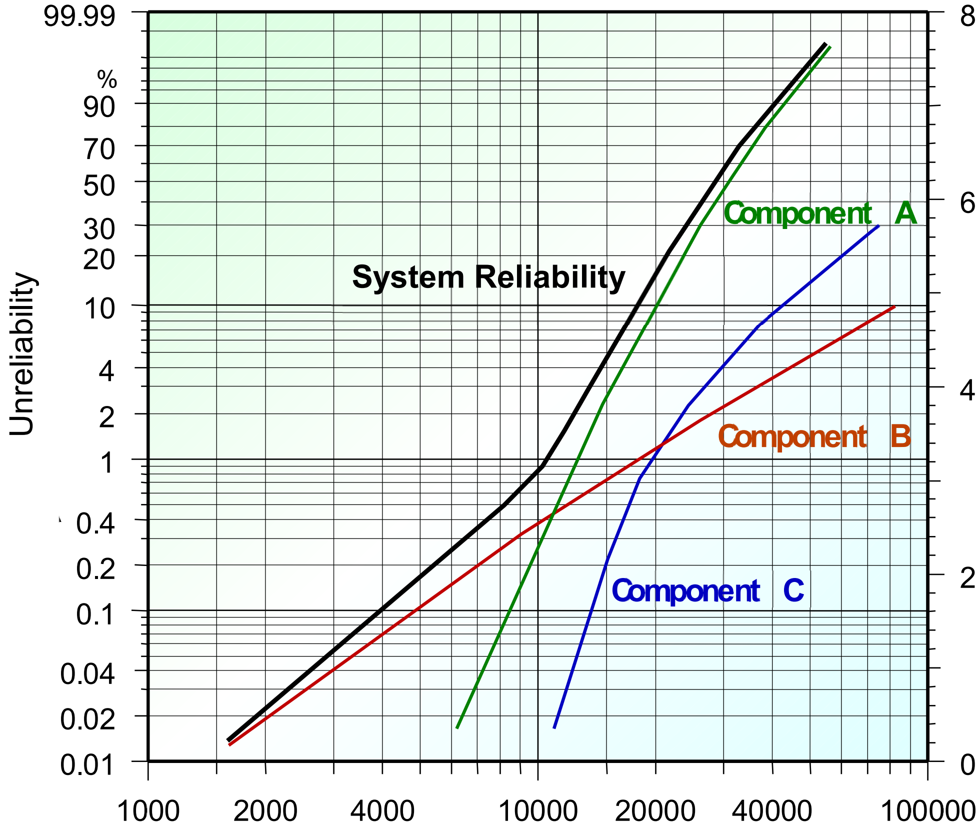
The Weibull parameters derived from the data record have a more or less large



deviation with respect to the specified b and T. It is advisable to generate a relatively large number of points and to then subsequently classify these data (depending on the required number of data items). Example: A Weibull curve is to be generated with $b \approx 2$ and $T \approx 1000$ as well as 500 data items. One sweep with classification results in e.g.

8. Reliability of the System

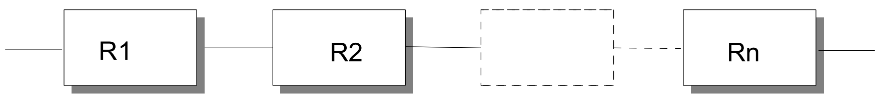
As a rule, the various components that make up a system have different failure rates. The overall reliability is generally defined as the "boundary line" of the individual components:



In the illustrated example, component C has a failure-free time. The component B dominates at the beginning, however, component A is decisive in the latter stages. Component B is to be improved accordingly for the purpose of improving the initial situation (warranty period). Component A is decisive for the further reliability over 10000 km.

! The overall system is therefore characterised by its weakest components. Corresponding to the "Pareto principle" it is necessary to concentrate on these components. Once the components in question have been decidedly improved, emphasis is shifted to the next weakest component. From an economics point of view the cost-benefit ratio is, of course, an additional authoritative factor.

In addition to these fundamental considerations, from a mathematical point of view, the overall reliability including several components or parts is derived as a function of the corresponding interaction of these components.
If the components are "configured in series", i.e. the functional chain) extends from one component into the next, the corresponding probability of survival R_{total} is derived from the multiplication of the individual probabilities of survival:

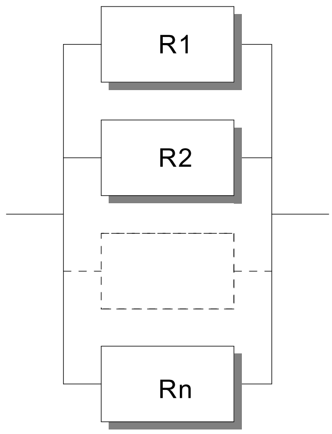


$$R_{total} = R_1 \cdot R_2 \cdot ..... \cdot R_n = \prod_{i=1}^{n} R_i$$

The system reliability is always poorer than the poorest individual component. The system reliability is reduced by each additional component. If a component fails, it immediately affects the entire system and the components need not necessarily be technically linked with each other. For example, a vehicle fails when either a tyre no longer has any air in it or when the engine fails. The serial configuration is therefore the basis for calculating the overall reliability. Since there is the relationship $H = 1 - R$ between the probability of survival R and the failure probability H, the following formula applies:

$$H_{total} = 1 - (1 - H_1) \cdot (1 - H_2) \cdot ..... \cdot (1 - H_n) = 1 - \prod_{i=1}^{n} (1 - H_i)$$

Components "configured in parallel" result in redundancy. The overall system fails only when all components have failed.



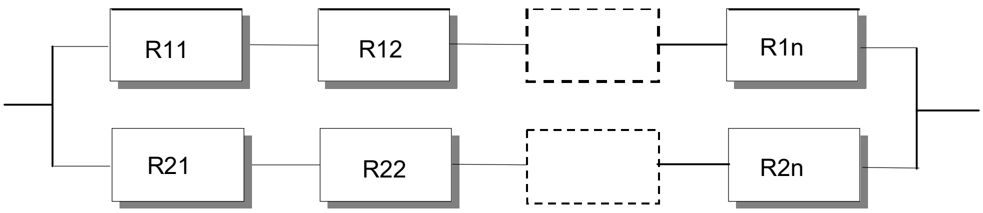
$$H_{total} = H_1 \cdot H_2 \cdot ..... \cdot H_n = \prod_{i=1}^{n} H_i$$

and the probability of survival

$$R_{ges} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

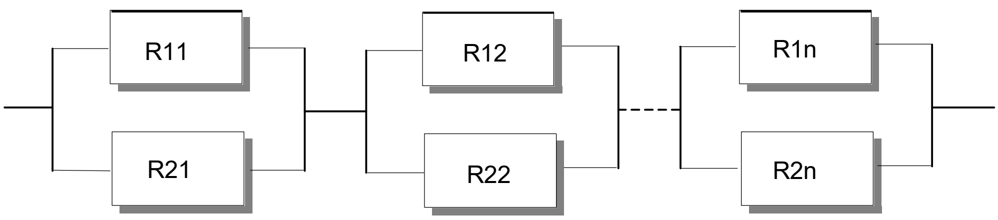
It is also possible to form combinations of components configured in series and parallel. The following probability of survival results from two serial "power trains" that are configured in parallel:

$$R_{total} = 1 - \left[1 - \prod_{i=1}^{n} R_{1,i}\right] \cdot \left[1 - \prod_{k=1}^{m} R_{2,k}\right]$$



A system fails when at least one component has failed within both series-connected power trains.

The following applies to several parallel circuits that are configured in series one after the other:

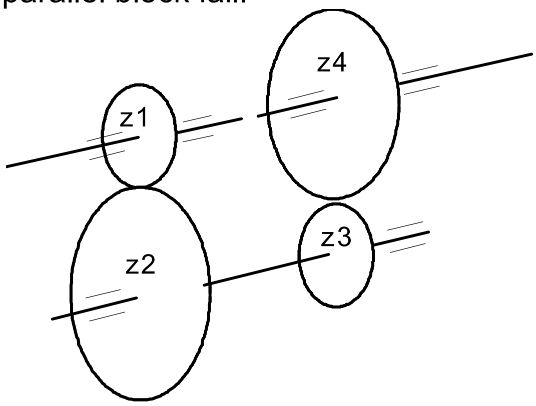


Here it is:

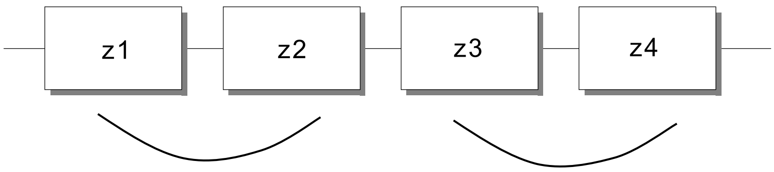
$$R_{total} = \prod_{k=1}^{n}\left(1-\prod_{i=1}^{2}\left[1-R_{i,k}\right]\right)$$

This system fails when both components of a parallel block fail.

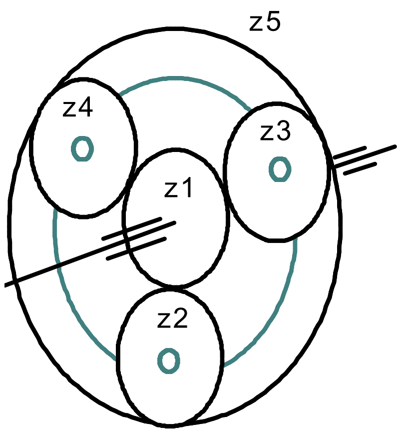
A typical example for series configurations in the following gear mechanism: The set up contains only series connections of up to 3 assemblies. However, the overall system can be determined by a 2x2 combination.

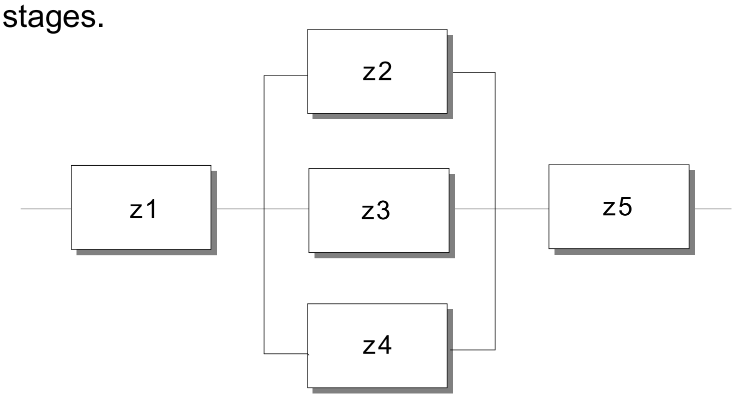


Serial force and function flow



Example of a planetary gear: Here, the illustrated components can be considered as subgroups that are defined in stages.



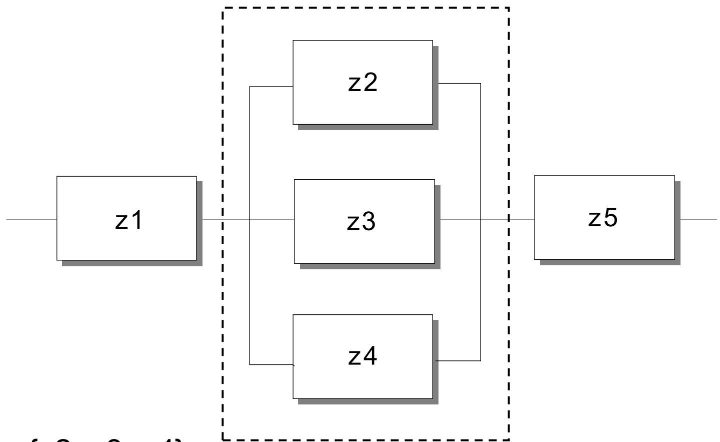


Power and function flow partly parallel, partly serial

R1= 0.85
R2=R3=R4 = 0.90
R5 = 0.80

R* = 1 – (1 - z2) (1 - z3) (1 - z4)
= 1 – (1- 0.9) (1- 0.9) (1- 0.9)
= 0.999

R_{total} = R1· R* · R5
= 0.85 · 0.999 · 0.8
= 0.679

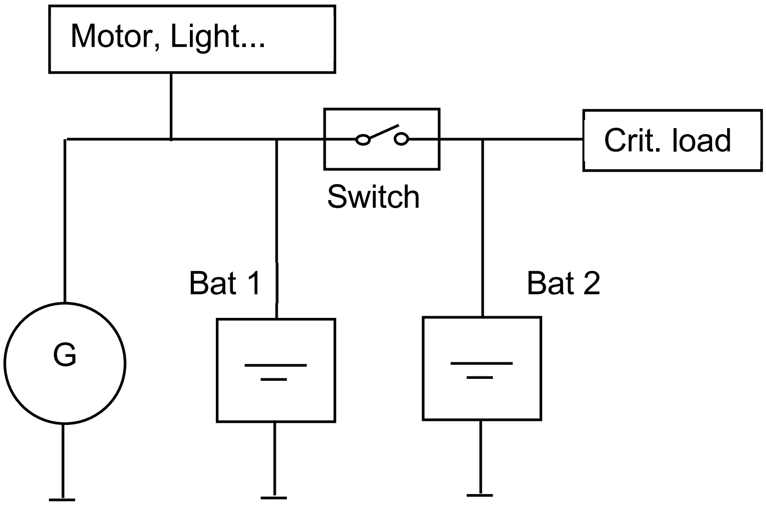


Subgroup: z* = {z2, z3, z4}

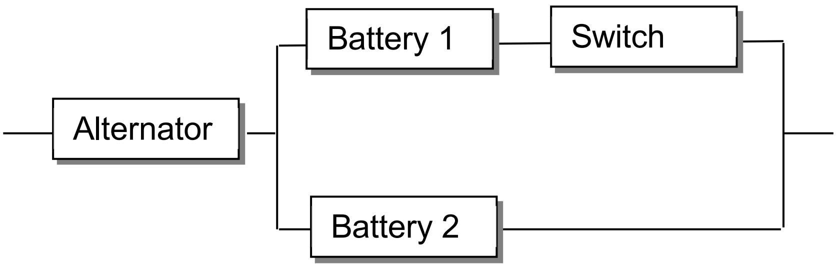
z*

Example of power supply

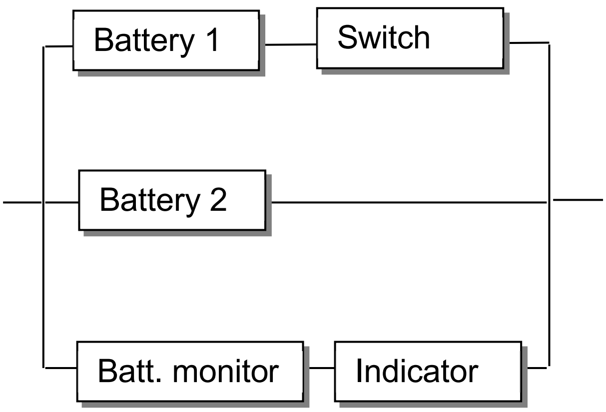
It is necessary to reliably supply one or several critical loads with power. Considerations: The auxiliary battery 2 is to provide backup in the event of the main battery 1 failing. A switch is to prevent battery 2 supplying the normal loads with power under certain conditions.



What does the block diagram for the availability of the critical loads look like?
What does the block diagram for critical loads relevant to safety look like with battery monitor and indicator?



Block diagram for system availability



Block diagram for safety equipment

With battery monitor and indicator.
Alternator alone is not sufficient for supplying critical loads with power.

It may be possible that the same element occurs several times, e.g. because a sensor supplies information for various evaluator units.
The block diagram should not be confused with the actual current flow in a circuit but rather should always be considered in terms of the availability or reliability of the components.

The greatest problem is obtaining the reliability figures of the individual components.

9. Fault Tree Analysis

The Fault Tree Analysis, briefly FTA, was developed originally in the sixties in the area of the American telecommunication industry and aircraft industry. Only in the beginning of the eighties a formalization of these methods occurred at the instigation of American authorities.

The Fault Tree Analysis is used for reliability- and safety security analyses. The proceeding is very similar to the Reliability Block Diagram RBD.

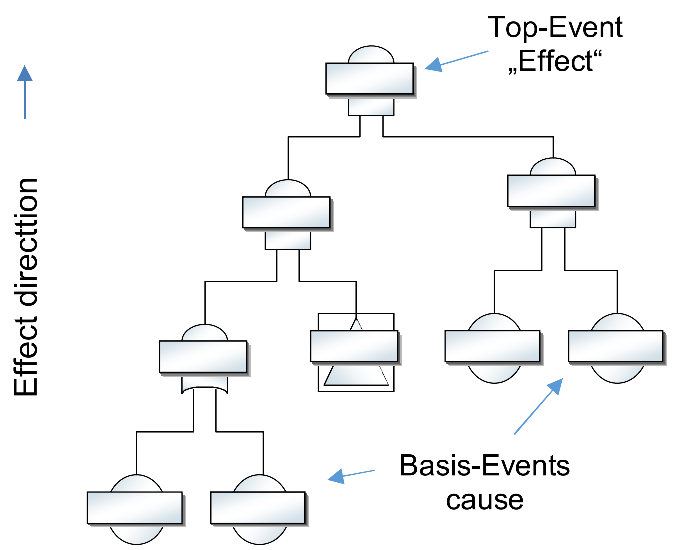
The aim is to determine possible combinations of causes which can lead to certain undesirable events (event), the so-called top-level events.

The goal is to identify possible combinations of causes that can lead to certain adverse events, the so-called top-level events. Furthermore, the task of an FTA is:

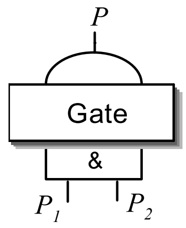
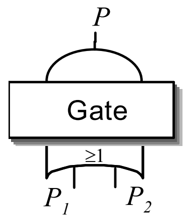
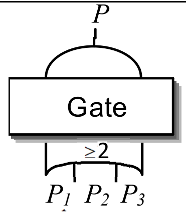
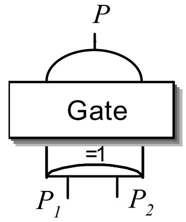
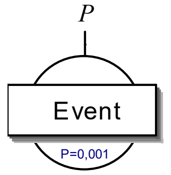
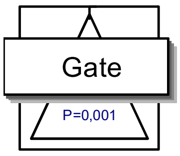
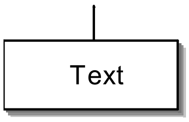
- The generation of a graphic / logical tree structure to the understanding of the connections.
- Identification of possible failure causes and their combinations.
- Calculation of the probability of the undesirable event. Comparison of variations.
- Comparison of variants

The link of the events is made in each case by different logical operators, the so-called gates. Device faults, operating faults and software faults which can cause with a certain probability to undesirable results belong to the events among other things.

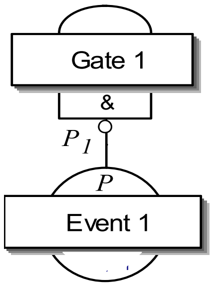
The following symbolism is used to the representation of the fault tree:



Symbolic

	&	**AND-Gate** The output event enters only if all input events apply. The output probability P is calculated with:
	≥1	**OR-Gate** The output event enters if one of the input events applies. The output probability P is calculated with:
	≥2	**Vote-Gate** The output event occurs when at least two input events occur. At least 3 inputs are required for this.
	=1	**XOR-Gate** (**Exclusive-OR**) The output event enters if only one of the input events apply but not both. The output probability P is calculated with:
		**Basic-Event** Primary base event or failure. The probability P is defined directly and mostly comes from manufacturer's data of the component. As with the reliability block diagram P is dependent on the time (component age).
		**Sub-Gate** At this point the other representation is interrupted. The given probability P represents the summary of other sub-elements which are not shown further.
		**Neutral text element** Text elements can be inserted in paths to show additional information, or to insert other "creases". At several entrances this element works like an OR gate.

Hint: The symbolism is according to country and standards, as well as in software partly differently.
The exit from P can be negated (example on the left). Then P describes the probability that the event does not enter. This is marked by a symbol of a circle at the entrance of the following gate.
The input probability of the following gate is: $P_1=1-P$.

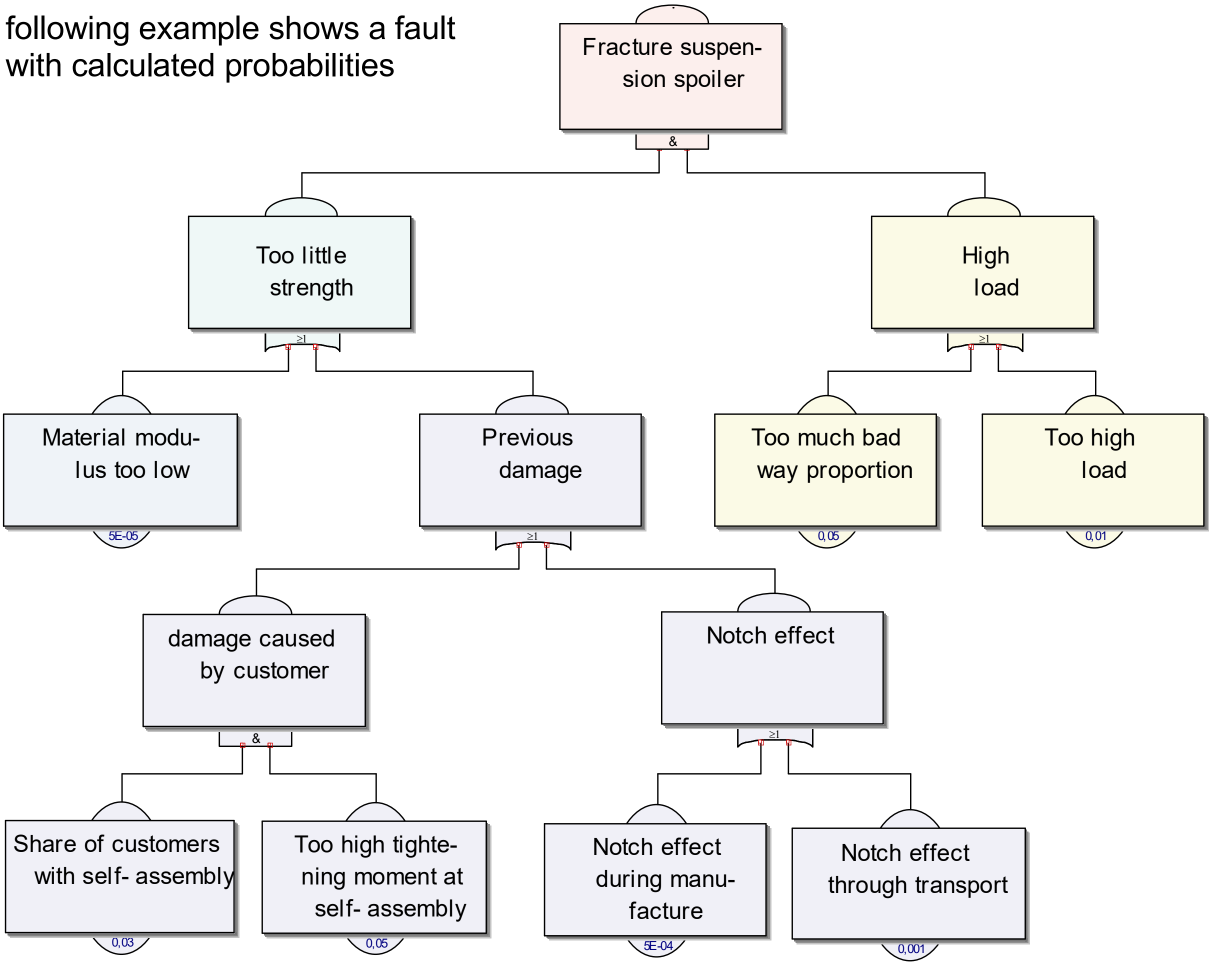


The advantage of the FTA is the hierarchical tree structure. In the upper area you can find the basic connections in the coarser area, in the lower area you can find the details. The FTA thus also provides good documentation of the relationships, even if the probabilities are not specified (qualitative fault tree).

There must be no opposing influences in the FTA, the elements must be independent of one another.

The fault tree starts at the top with the Top Event. At the bottom, the event paths are broken down further and further until one arrives at the Basic Events, or at the Sub-Gates, for which the further details are not detailed.

The following example shows a fault tree with calculated probabilities



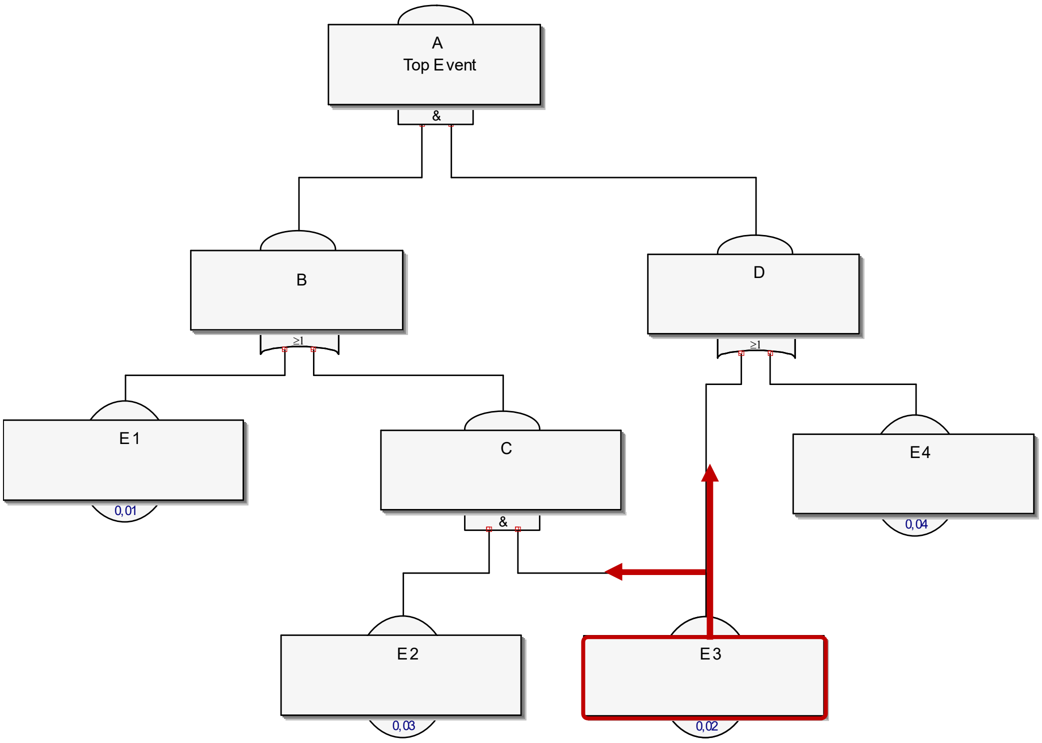
In this example, the system is initially divided into its properties (strength) and what it experiences (load). This is followed by the relevant connections and, at the end, the actual causes (basic events).

In addition to the example shown, the FTA is particularly suitable for the electrical / electronic area or for control and regulation systems. The FTA is not the right tool for just looking at software.

In the comparison no probability is treated moreover in a causes-effect diagram. One looks only at the "critical" moment when the fault occurs as a top event.

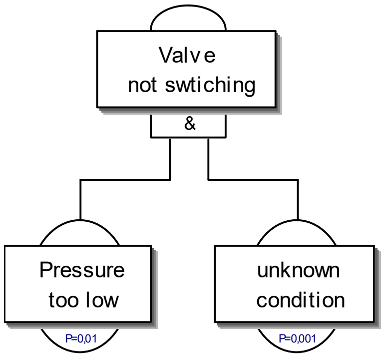
Calculation via so called Cut Set

If Basic Events or Sub-Gates have an influence on several gates, a calculation starting from the base event will give incorrect results for the top event. The following example shows such a situation in which event E3 has an influence both in gate C and in gate D (e.g. a temperature could affect several paths):



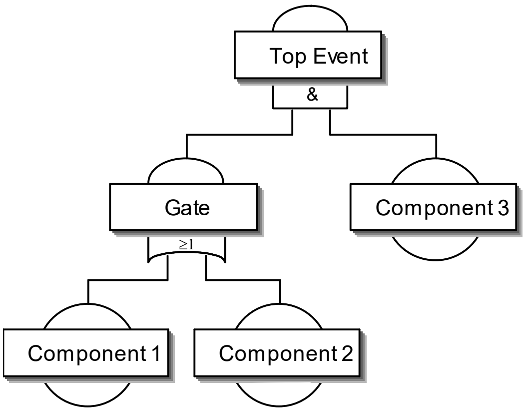
The calculation according to the algorithm of J.B. Fussell takes place step by step in tabular form until only Basic Events or Sub-Gates remain.

The complete production of the fault tree assumes that one can name all components and conditions. This is not always given under circumstances if, e.g., a failure appears as only temporary and is not known what condition must be still given here. This can be avoided first by a "place holder" who is to be determined later

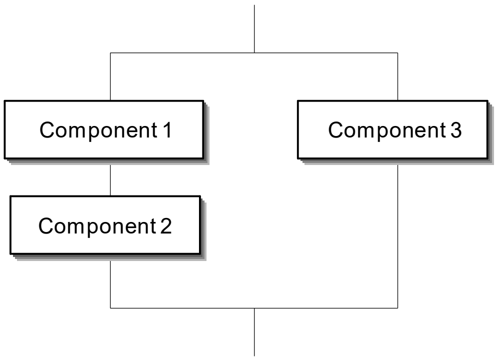


Comparison with a Reliability-Block-Diagram (RBD)

As at the beginning described, the comparison with the reliability block diagram should be still shown (Reliability-block diagram = RBD).
Conspicuously is that here no gates are shown, but only the suitable components which are the base events in the FTA.



Fault - Tree



Reliability Block Diagram

While in the FTA a redundancy is shown merely as AND link, this seems more striking in the RBD by the parallel arrangement. The difference between OR / AND link is stronger emphasised here graphically. The advantage is also that here less elements are needed. However, the disadvantage of the RBD is that none Exclusive OR-links are possible.

With the treatment of a problem, the comparison with a cause-effect-diagram is also looked often. If there are "conditions" or components which are not known yet, exists in the FTA the problem to name this. Here as a rule, one places pseudo-elements which are to be determined even closer. In the cause-effect-diagram one is led there about the physical / technical active chain rather on the still missing connections. Further details to active diagram are described under:

www.weibull.de/COM/Systemanalysis.pdf.

10. Reliability Growth Management (Crow AMSAA)

One understands by a Reliability Growth Management the improvement steps within the development of components. While component variations are not allowed in the Weibull analysis, a change of the test objects is possible here. Besides this, a small sample size and different failure modes can be shown together. Often it is the purpose to identify the developing progress and to make a forecast.

The Reliability Growth Management was developed in the 60s by Duane by General Electric Motors Division for military systems. An essential improvement and extension of the Duane proposal was developed by L.H.Crow at the U.S. Army Material System Analysis Activity (famously better known as Crow-AMSAA model).

The connection between accumulated test duration t and the accumulated failures $N(t)$ is:

$$N_{(t)} = \lambda\, t^{\beta}$$

with λ = shape parameter (not to mistake with same name in the chapter failure rate)
β = slope parameter (not to mistake with the Weibull-slope parameter)

In a double-logarithmic diagram there can be created a straight line with the slope β. The mean failure rate is:

$$\rho_{(t)} = \lambda\,\beta\, t^{\beta-1}$$

The mean time between failures $MTBF$ is calculated with:

$$MTBF = \frac{1}{\lambda}\, t^{1-\beta}$$

Here it was indicated, that if the form parameter or the slope is $\beta \neq 1$, it concerns to a non homogenous Poisson process (NHPP). That means if $\beta > 1$ there rises the failure rate or the failures come earlier and if $\beta < 1$, then there decrease the failure rate and the failures come slowly. For the case, that the form parameter is $\beta = 1$, one say it is a homogeneous Poisson process. In the development phase there should be the slope $\beta < 1$, which is an indicator for reliability growth.

The best known methods to determine the parameters are:

1. Least square estimation (regression analysis)
2. Maximum Likelihood estimation (MLE, or MIL-HDBK-189)
3. Power Law (*NHPP* or unbiased MLE)

For the least square estimation, the cumulated test times and the cumulated number of failures have to be used as logarithm values. Then the standard method least square has to be applied. The least square estimation is suggested for small sample sizes less then 5.

The parameter λ and β are determined with the method **Maximum Likelihood** through the following formulas:

Type-I (after a defined test time the test is finished)

$$\beta = \frac{n}{n\ln(T) - \sum_{i=1}^{n}\ln(t_i)} \qquad \lambda = \frac{n}{T^{\beta}}$$

where is T = defined total test time.

Type-II (after a defined number of failures the test is finishes)

$$\beta = \frac{n}{(n-1)\ln(T) - \sum_{i=1}^{n-1}\ln(t_i)} \qquad \lambda = \frac{n}{t_n^{\beta}}$$

For the **Power Law Model** (NHPP described in IEC 61164) there has to be used the following formulas:

Type-I (time truncated)

$$S = \sum_{i=1}^{n}\ln\frac{T}{t_i} \qquad \beta = \frac{n-1}{S} \qquad \lambda = \frac{n}{T^{\beta}}$$

T = defined total test time , t_i cumulated test time for each failure

Type-II (failure truncated)

$$S = \sum_{i=1}^{n}\ln\frac{T_n}{t_i} \qquad \beta = \frac{n-2}{S} \qquad \lambda = \frac{n}{T_n^{\beta}}$$

T_n = total test time with defined number of failures, t_i cumulated test time for each failure

The least square estimation does not make a distinction between type I and type II. As best estimation the power law model is often regarded.

For example there are the following test times:

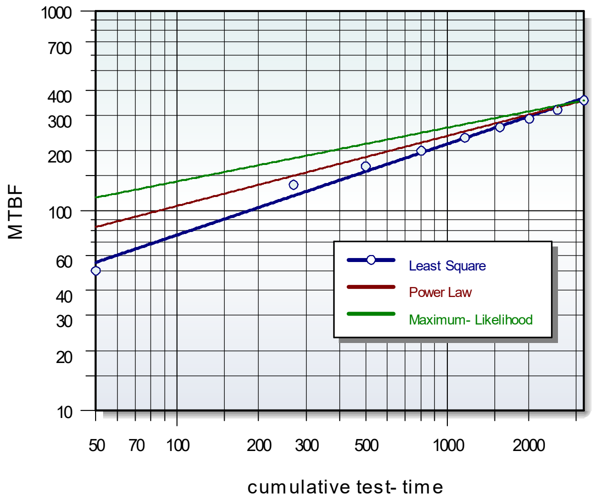
N	test time t	cumulated T	$MTBF$
1	50	50	50,0
2	220	270	135,0
3	230	500	166,7
4	300	800	200,0
5	360	1160	232,0
6	410	1570	261,7
7	450	2020	288,6
8	540	2560	320,0
9	650	3210	356,7

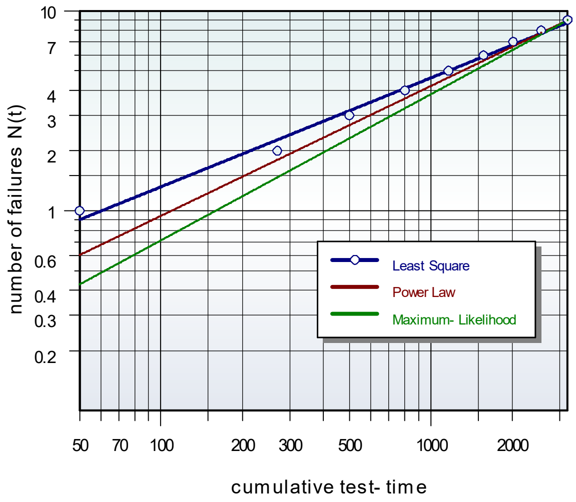
$MTBF$ is calculated through:

$MTBF = T/N$

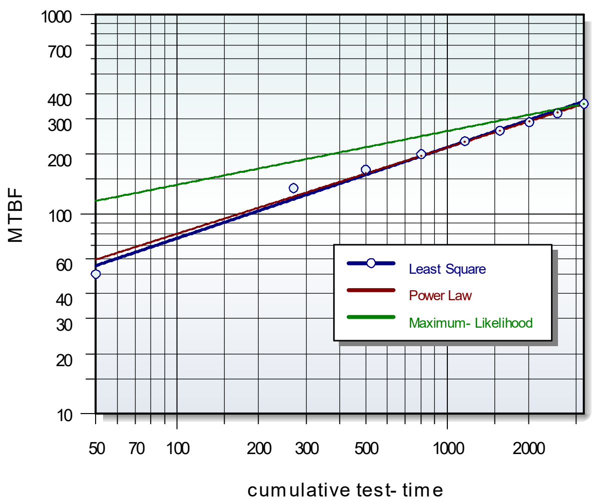
After the 3 methods the can be compared $MTBF$ and $N(t)$ in logarithmic axes:

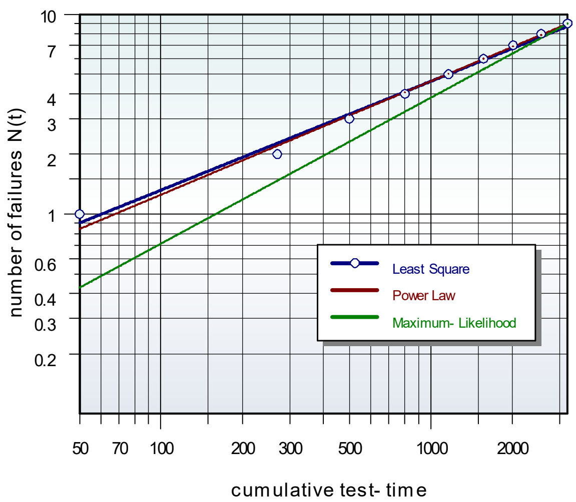
Type-I (time truncated)





Type-II (failure truncated)





In the comparison the methods for type I give relatively great differences. Typically the Maximum Likelihood method lies farthest away from the points. For type II the power law model approaches to the least square estimation well. Depending on the position of the right points it is recommended to extrapolate with the power law model or the straight line.

The confidence area for $MTBF$ can be calculated with the χ^2-distribution:

$$MTBF_u = \frac{2t_i}{\chi^2_{\alpha/2,2n}} \qquad MTBF_o = \frac{2t_i}{\chi^2_{1-\alpha/2,2n+2}}$$

Accordingly a confidence area is given for the cumulative number of failures with:

$$N_{(t)u} = \frac{t_i}{MTBF_u} = \frac{\chi^2_{\alpha/2,2n}}{2} \qquad N_{(t)ó} = \frac{t_i}{MTBF_o} = \frac{\chi^2_{1-\alpha/2,2n+2}}{2}$$

Both formulas concerns to the least-square method.

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11. Service life prognosis from degree of wear

There is often not enough time to wait for a component to fail. If it is possible to measure the "degree of damage“, there is a chance to extrapolate to the remaining service life. Assuming the wear is proportional to the distance covered (mileage), it is possible to use a rule of three to make a projection.

$$t = \frac{t_1}{\Delta D} D_{rest} + t_1$$

where t : Expected service life

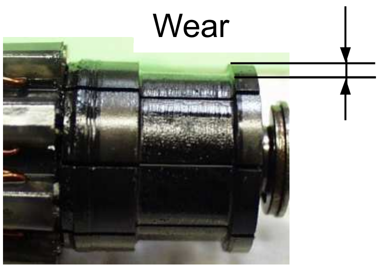
t_1 : Moment at which the degree of wear was measured

ΔD : Degree of wear, e.g. brake pad thickness when new – remaining brake pad thickness

D_{rest} : Remaining brake pad thickness available

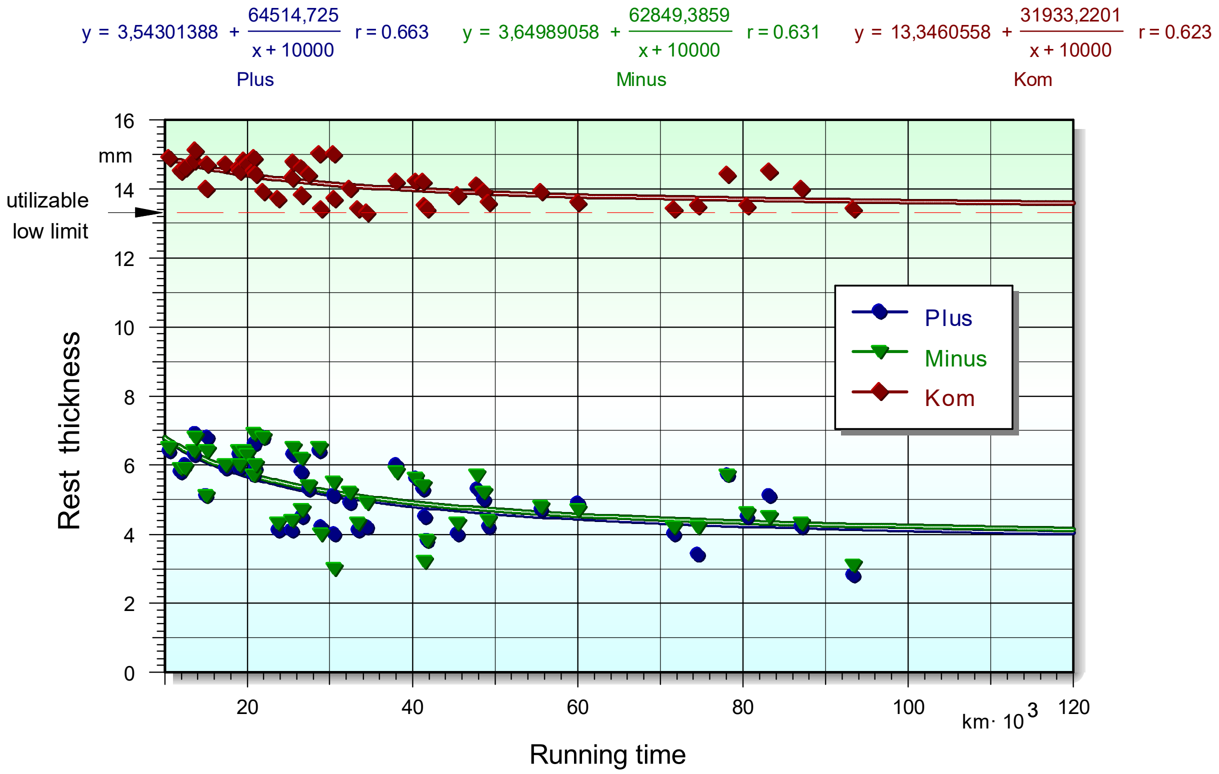
The calculated service life values t can be represented in the Weibull plot. In many cases, a linear wear characteristic cannot be assumed. Initially, the progression should be represented as a function of time from the observed wear values and a suitable functional approach selected (e.g. e-function).

Example: A new pump was tested as part of large-scale trials. 47 vehicles were equipped with the pump. A deduction with regard to the expected service life was to be made after a defined period of time. The precondition was that the pumps covered a most diverse range of distances as possible as the initial task is to determine the wear as a function of the running time. All pumps were examined. All components that determine the service life, i.e. commutator, positive and negative poles (carbon elements) were measured with regard to their degree of wear.



Commutator

The effective range of the commutator extended up to a minimum diameter of 13.4 mm. The positive and negative carbon brushes were worn when a residual thickness of 0 was reached. Evaluation of the measurements resulted in the following representation as a function of distance (mileage):



Despite the large scatter or dispersion range, a non-linear relationship can be clearly seen. This means the wear is digressive over distance. This is attributed to the fact that the spring force and therefore the contact force of the positive and negative carbon brush on the commutator decreases with wear. Using a suitable functional approach, attempts were now made to conclude the projected service life. The function

$$Y = a' + \frac{b'}{x + c'} \qquad \text{->} \qquad D = a' + \frac{b'}{t + c'}$$

proved to be the best approach (see equations above the diagram). Y is the residual thickness D and x the distance covered (mileage) t. The "offset“ c' was defined in terms of the lowest km value of 10000 km (vehicle with low mileage).

Using this approach, the progression of the curve is to be determined for each pump (the coefficients a' and b'). The coefficients are determined based on a starting point at 0 km and the distance covered by the vehicle (mileage). This gives:

$$D_{start} = a' + \frac{b'}{0km + 10000km}$$

In the moment of measuremt, there is:

$$D_1 = a' + \frac{b'}{t_1 + 10000km}$$

D_{start} is 15.2 mm for the commutator and 7 mm for the positive/negative carbon brushes (new condition). These two relationships were now equated and resolved for b

$$b' = \frac{D_{start} - D_1}{1/10000km \; - 1/(t_1 + 10000km)}$$

and a'

$$a' = D_1 - \frac{b'}{t_1 + 10000km}$$

The required **theoretical service life** is then calculated for each pump individually with the useable min. diameter of D_{min} = 13,4mm:

$$t = \frac{b'}{D_{\min} - a'} - 10000km$$

The service life values projected in this way are transferred to the Weibull plot. From a purely mathematical point of view, there are values that will not reach the lower critical wear point (x value negative). This means that the end of the service life is beyond the service life of the vehicle. These pumps cannot be represented in the plot, however, they must be taken into account in the scope of the sample (referred to 47 pumps).

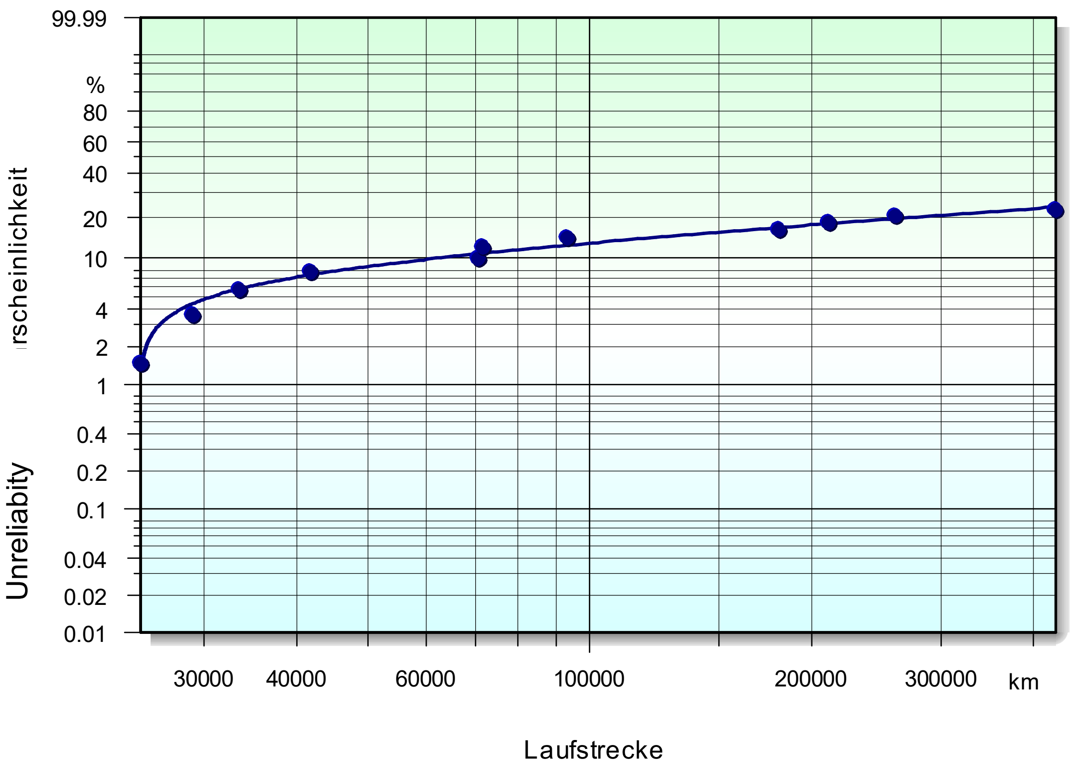
In terms of failure probability, the positive and negative carbon brushes are normally included in the system reliability of the pump (block diagram – serial configuration). This is represented by:

$$R_{ges} = R_{Plus} \cdot R_{Minus} \cdot R_{Kom}$$

or

$$H_{ges} = 1 - (1 - H_{Plus}) \cdot (1 - H_{Minus}) \cdot (1 - H_{Kom})$$

However, the resulting service life values for the positive and negative carbon brushes were considerably higher than those of the commutator. In simplified terms, the service life of the pump could therefore be described with the service life of the commutator alone, resulting in:



As expected for wearing components, this resulted in a failure-free period t_o. The shape parameter b <1, however, signifies early failures. The apparent contradiction is attributed to the fact that the critical pumps (only 11 out of 47) are subject to a tolerance or production-related random influence which is of significance in this case.

The result of the investigation found that the new pump is not suitable and the production influences are to be examined to determine the reason.

12. Consideration of failures not yet occurred

Sudden death

If specimens are removed from a service life test before they have failed, this random test will be considered incomplete. In this case, it is clearly incorrect to enter the corresponding "running times“ with the ordinal i in precisely the same way as if they have failed. Simply omitting them from the entire consideration would mean that important information which could have been evaluated is not included. The use of this information, indicating that a number of parts has reached a certain service life without failing, is referred to as "sudden death testing“).
In the laboratory it is often possible to test several specimens simultaneously on the same test set-up or apparatus. If one of the parts fails, in most cases, the others are still in working order. These parts are still included in the subsequent evaluation although they are not left to run to the end of their service life. Assume, for example, that the following tests were carried out using 3 specimens at the same time, all on the same test set-up or apparatus.

Running time in h	Number of failures	Number of parts without failure
10	1	2
14	1	2
16	1	2
18	1	2

The first failure is assigned the rank number 1. Although the two parts without a failure are not represented directly in the subsequent Weibull diagram, they will indirectly influence the frequency value of the subsequent failures. The rank number 2 is not given to the failure at 14 h but rather a value greater by a delta value. This value is calculated using:

$$\Delta = \frac{n+1-Rank_{(i-1)}}{1+n_{Next}}$$

where n_{Next} is the number of subsequent test specimens. Entering n = 12 results in:

$$\Delta = \frac{12+1-1}{1+9} = 1{,}2$$

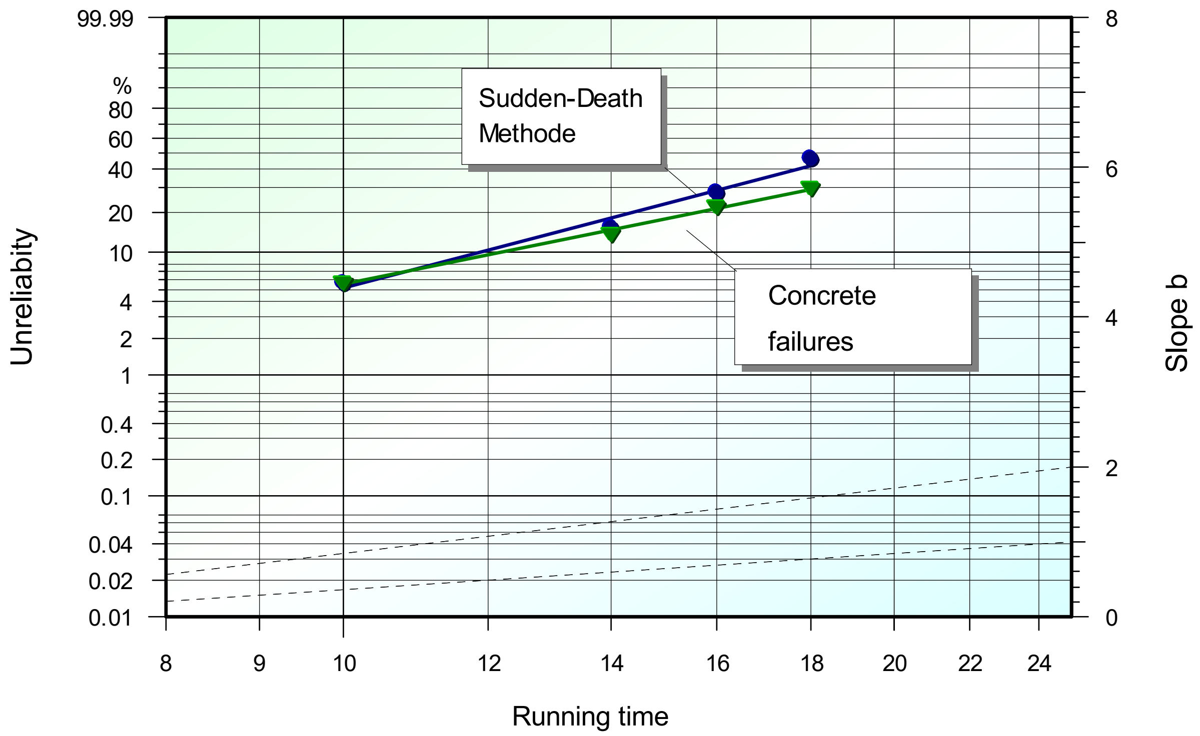
and the $rank_{(i)} = rank_{(i-1)} + \Delta = 2.2$. If the next test specimen also fails, the next rank assumes the value of the previous rank plus the currently determined delta. In this example, the next failure is at 16 h with the new delta of

$$\Delta = \frac{12+1-2{,}2}{1+6} = 1{,}54$$

resulting in $rank_{(i)} = rank_{(i-1)} + \Delta = 3.74$ and so on.
The associated frequencies are defined by:

$$H = \frac{Rank_{(i)} - 0.3}{n+0.4} 100\%$$

results in the following representation:



The sudden death method results in a steeper slope than the observation method which takes into consideration only the pure failures (with n=12). If, in practical applications, all test specimens were subjected to a test individually and the failures correspondingly plotted, the result would basically that of the sudden death method. The advantage is the considerably shortened test phase.

Evaluating data of defective and non-defective parts

If units have not yet reached a certain running distance, where others have an complaint, incomplete data arise like as in Sudden Death, but not necessarily in groups. Initially, all running times are sorted one after the other irrespective of whether they are defective or not. Example:

Ordinal	Distance*1000	Defective	Not defective
	40		x
	51		x
1	54	x	xx
2	55	x	
	59		x
	60	x	x
3	60		x
	61		x
4	62	x	xx
	etc.		

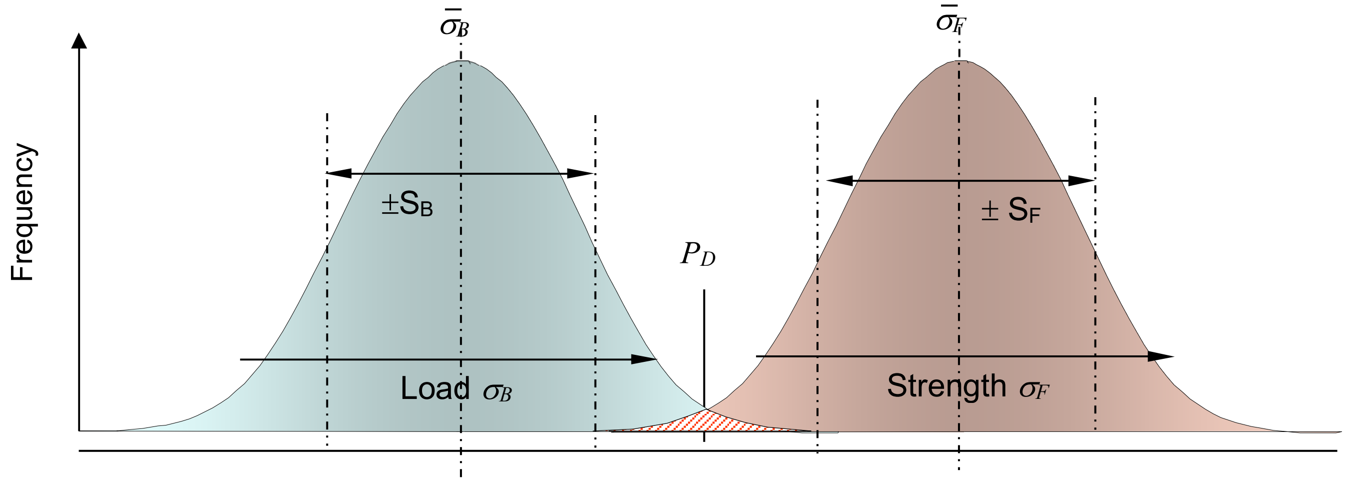
The non-defective parts are assigned to the next higher or equally high damage case. The ordinal refers only to the defective parts. If, at the end, only non-defective parts occur, they cannot be assigned to a distance value. They do, however, correspondingly increase the scope of n. Finally, the procedure is the same as for the sudden death method, the only difference being that the number of previous parts is calculated differently. In the sudden death method, the number of previous parts is 0 at the start and 2 in the example. In sudden death, the mean ordinal always begins with the number of the defective parts whereas this method additionally takes into account the non-damaged parts.
In contrast to Sudden Death method the necessity already arises for the first failure point for a correction, because this is incomplete. Instead of 1 the rank number becomes accordingly higher:

$$Rank_1 = \frac{n+1-0}{n+1-n_{before}} > 1$$

Number of non faulty parts before the first failure

13. Tests with normal load

Components and their tolerances (illustrated on right) are normally designed such that they have a corresponding safety allowance (safety factor = $\overline{\sigma}_F / \overline{\sigma}_B$) with respect to the load (illustrated on left).

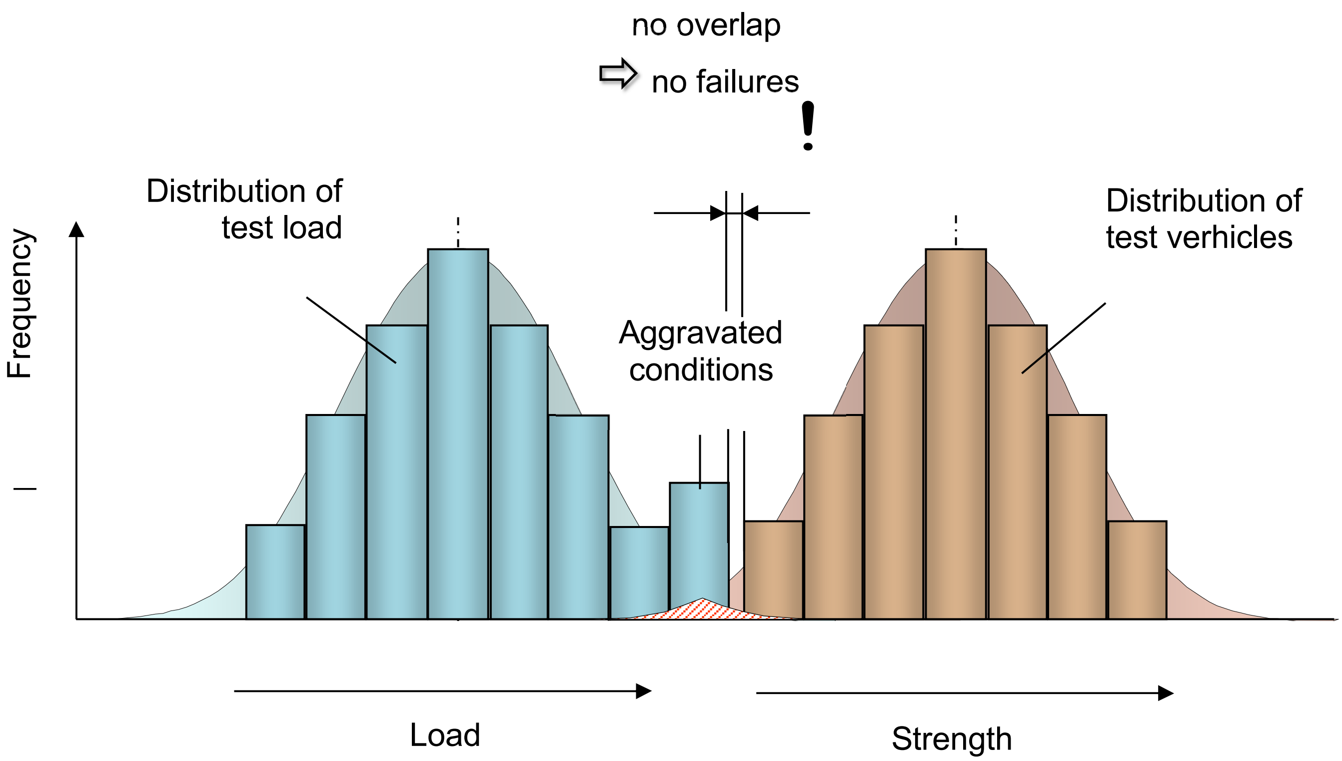


There is an unavoidable overlap when the random samples of both distributions are considered as unlimited. This overlap corresponds to the number of "defects" or represents the failure probability P_D. A lower component tolerance results in a smaller overlap. P_D is calculated with:

$$P_D = 1 - \Phi\left(\frac{\overline{\sigma}_F - \overline{\sigma}_B}{\sqrt{s_F^2 + s_B^2}}\right)$$ where Φ = Normal distribution for μ = 0 and s = 1

Strictly speaking, the calculation refers to components in the fatigue strength range. When used for the strength range for finite life, the deduction is to be referred to a certain load time or a number of load cycles.

A certain number of vehicles, for example is tested as part of the trials. The tolerances and fundamental design layout of the parts or components used exhibit a normal distribution in terms of their strength. Since the sample is a limited number random sample, the limits will be reached only at a certain value. The load cycles used in the trails also exhibit a normal distribution, however, more stringent conditions are simulated by additional driving cycles.



No failures occur in view of the still sufficient "safety margin". However, further tightening of the test conditions in the vehicle is not possible. The question is therefore posed how many components will fail in the entire production (constant progression of the distribution in the background). This results in an overlap of the lowest component strength with the highest-occurring load.
The following section illustrates how a corresponding statistical deduction can be made.

14. Tests without failures - Success Run

Minimum reliability and confidence level

In order to be able to draw conclusions with regard to the reliability of a component or assembly, tests are conducted with a limited number of test samples prior to actual series production. This is a relatively reliable method of discovering fundamental design flaws or manufacturing faults. On the other hand, the probability of determining faults that occur randomly or at low frequency is low if a considerably higher load cannot be applied in the test. This is generally the case in vehicle tests, in contract to the special component tests conducted on component test rigs or in the laboratory, permitting an increase in load by a factor of 2 and higher.
The initial question is how high is the probability P_A that a test specimen fails during the test:

$P_A = 1 - R_t^n$ where R_t = Reliaiblity at test time t for a test specimen; n = Number of test specimen

Rearranging the formula:

$$R_t = (1 - P_A)^{1/n}$$

The reliability for the test time t is calculated using:

$$R_t = e^{-(t/T)^b}$$

A reliability R_a: applies for the defined service life t_a:

$$R_a = e^{-(ta/T)^b}$$

Equating the two relationships and defining $Lv = t / t_a$ results in:

$$\frac{R_t}{R_a} = \frac{e^{-(t/T)^b}}{e^{-(ta/T)^b}} \quad \Rightarrow \quad \frac{\ln(R_t)}{\ln(R_a)} = \frac{-(t/T)^b}{-(ta/T)^b} = Lv^b$$

Consequently:

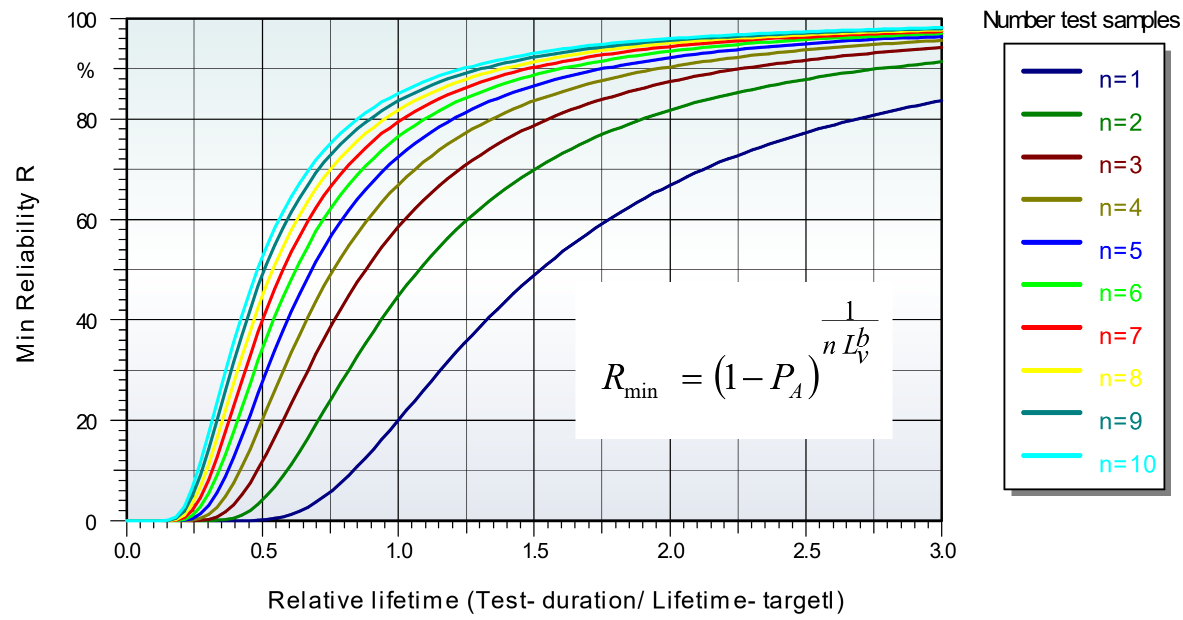
$$\ln(R_t) = \ln(R_a)\, Lv^b$$

$$R_t = R_a^{Lv^b}$$

Together with the number of test specimens $R_t = (1 - P_A)^{1/n}$ and equating results in:

$$R_t = R_a^{L_v^b} = (1 - P_A)^{\frac{1}{n}}$$

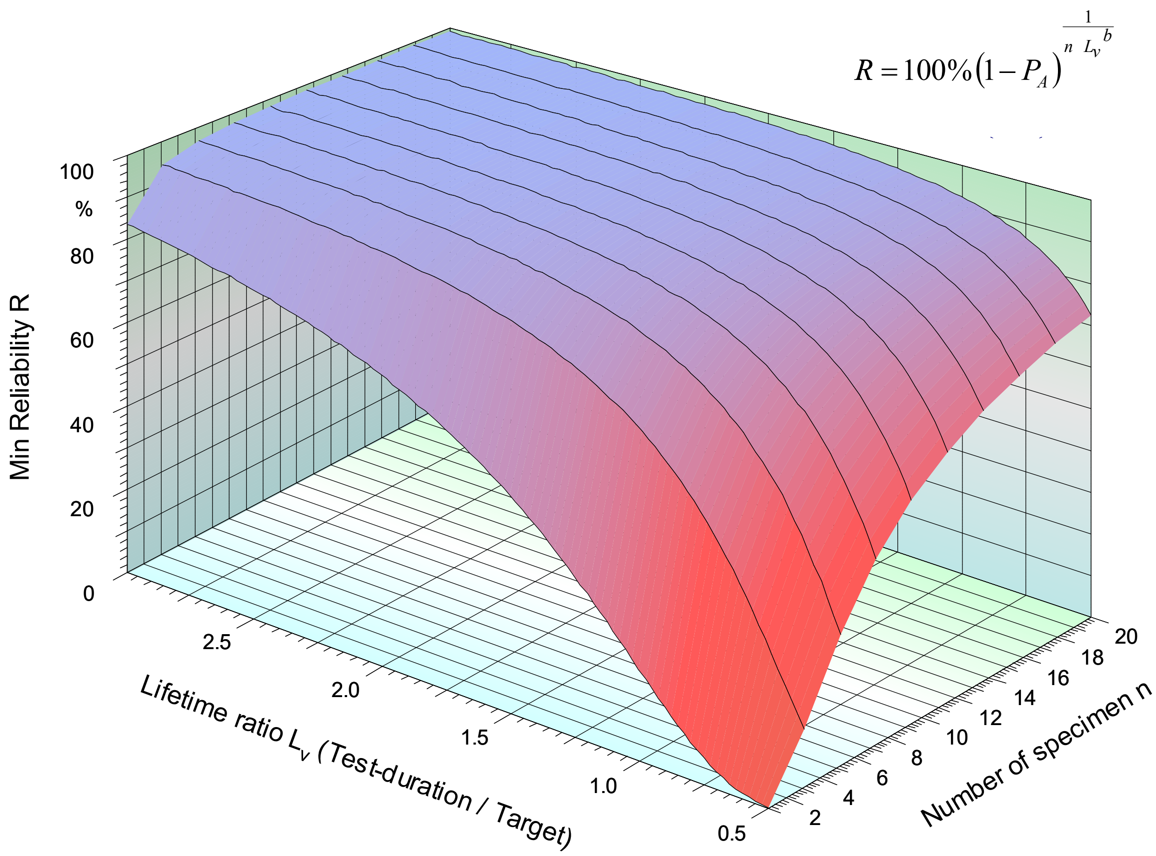
$$R_a = (1 - P_A)^{\frac{1}{n L_v^b}}$$



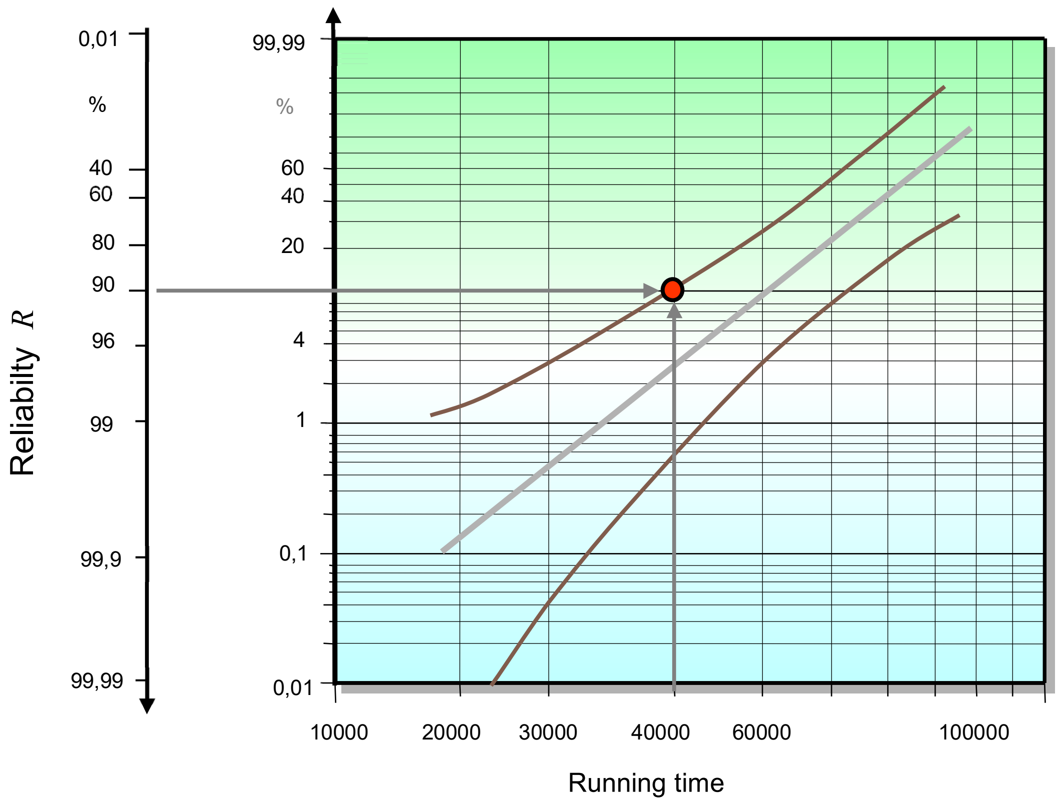
The reliability R_a is to be considered as the "guaranteed minimum reliability".

Example: The following representation is achieved for a confidence level P_A=0.8 and an estimated b=2.

In the 3D representation it can be seen that a further increase at the level of L_v and n provides no further decisive advantages.



On examining these considerations in the Weibull plot the following representation is obtained (P_A = 0.80 > upper confidence bound 90% t = 30000 R_{min} = 90%)



P_A was originally introduced as the probability for the failure of a test specimen. The prerequisite is that these test specimens are a random sample of a "population". A conclusion is drawn based on the samples where the definition corresponds to the confidence bound of the illustrated Weibull plot.
The calculated minimum reliability is not valid if the test specimens are "hand specimens" or prototypes with their manufacturing process not corresponding to subsequent series production.

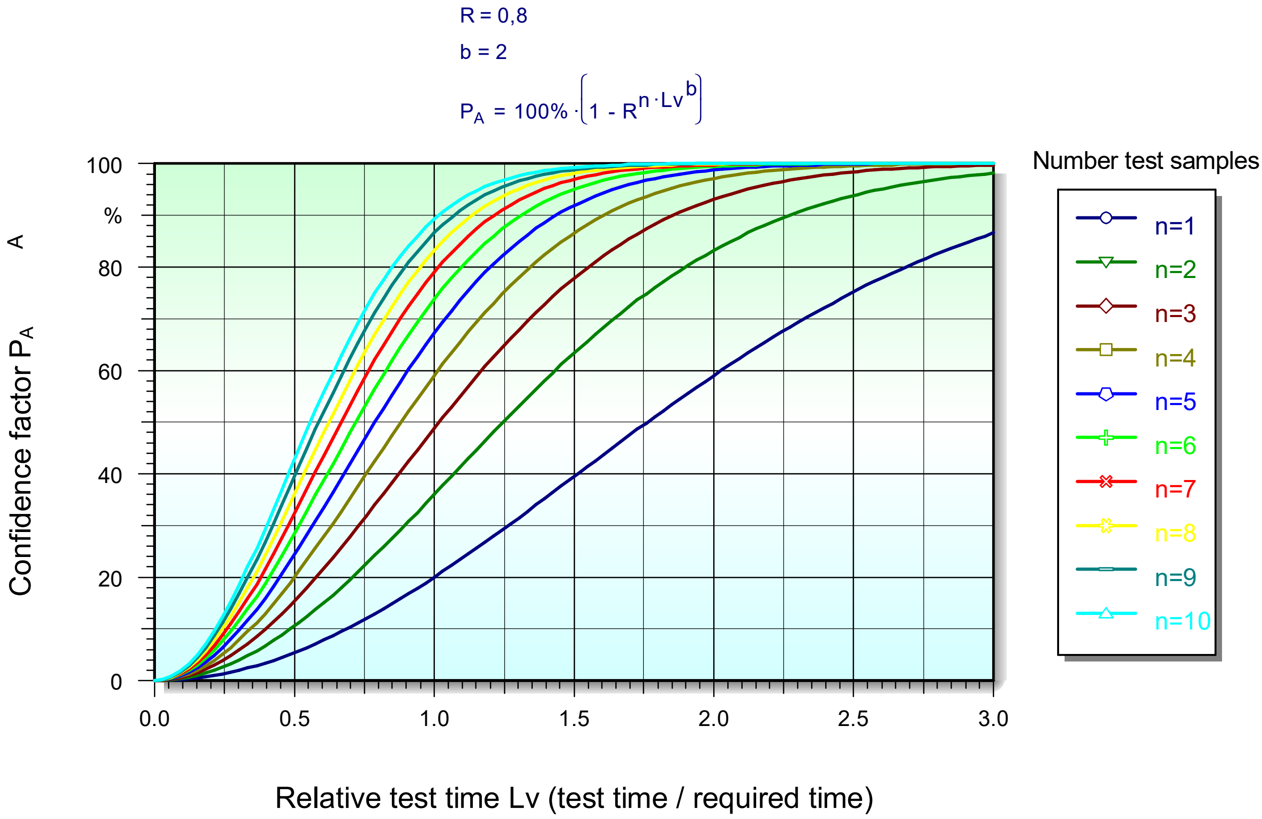
It should be noted that a lower b results in a lower minimum reliability. Initially, this is not to be expected as a low b results in a lower slope on the Weibull plot and therefore a higher failure frequency. This effect is caused if the target running time is less than the test time and one is moving to the left with a flat slope.

It can generally be assumed that for the confidence level or for determining the reliability, it is better to test less samples size for a longer time than many samples for a relatively short test time. On the other hand, with fewer samples the conclusion concerning the component scatter is also less reliable (minimum number of samples).

Lv should always be greater than 1 if the load cannot be increased. Irrespective of the mathematical minimum reliability, no part must fail at Lv<1 (minimum requirement).

If conclusions are to be drawn with regard to the reduction in service life to higher load, tests with concrete failures will be necessary, represented in a stress-cycle (Woehler) diagram.

If a certain minimum reliability is defined and the question is what confidence level is reached, the above formula is to be correspondingly rearranged to result in b=2 and R=80%:

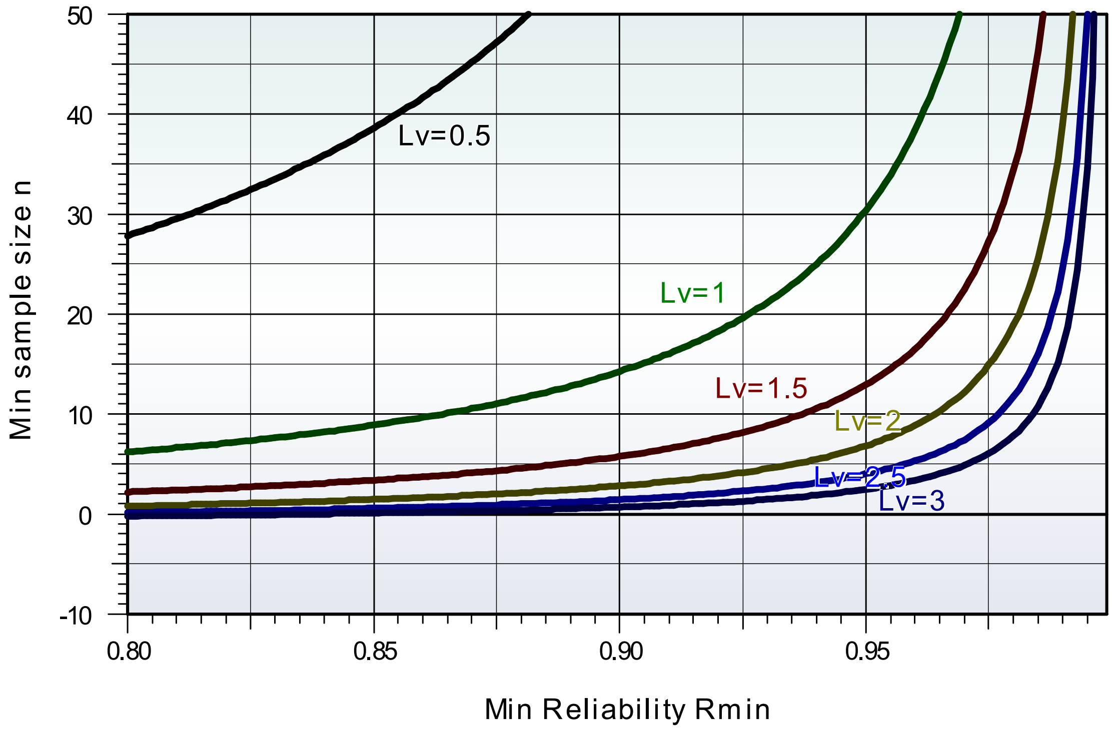


Minimum number of samples for tests

It is often necessary to determine the minimum number of test specimen for the purpose of verifying the defined reliability. However, there is no generally defined procedure for this purpose. In accordance with VDA, the necessary minimum number is calculated by transposing the formula for n:

$$n = \frac{1}{L_v^b}\left[\frac{\ln(1-P_A)}{\ln(R)}\right]$$

For P_A=0.80 and b=2 this results in:



As illustrated, the prerequisite for this scope of random samples is that no failures occur.
The procedure for determining the confidence bound can be used for establishing the minimum number of samples, resulting in the same consideration as when using a defined confidence level.

Example: The number of components to be tested is to be found if a double test time compared to the required service life is possible and a minimum reliability of R=90% is required. No parts fail during the test. This results in n=3 for a confidence level of P_A=0.80.

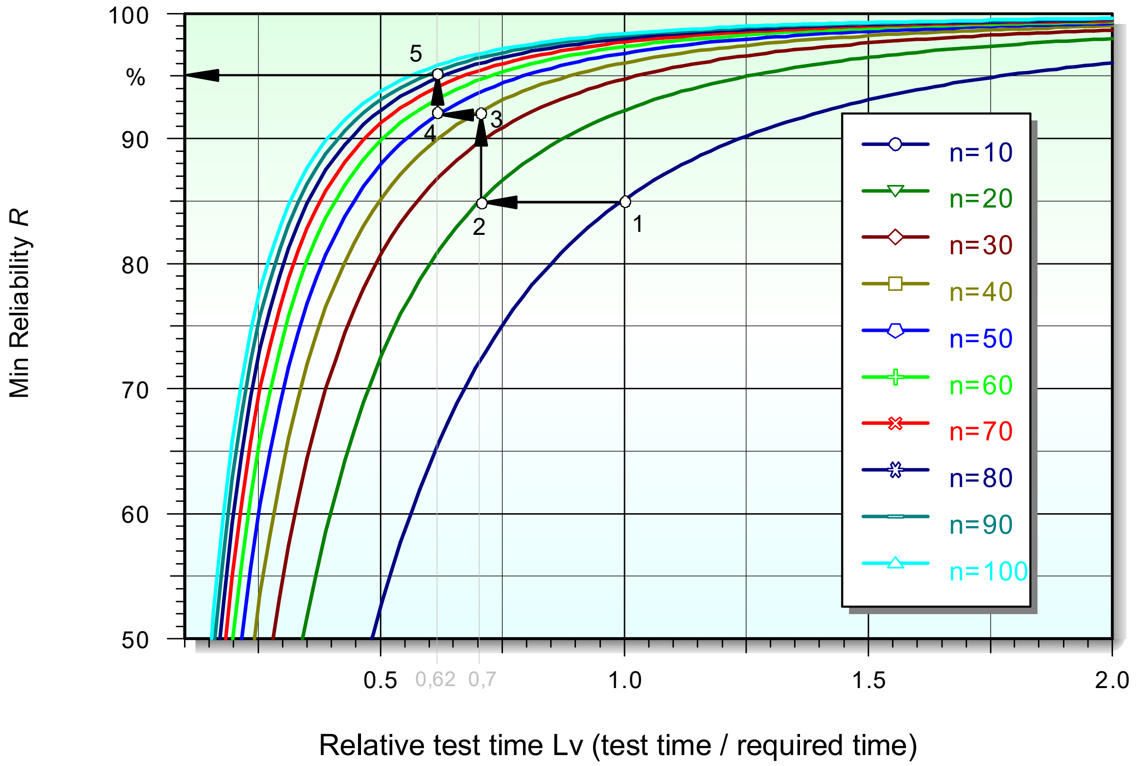
Determining minimum reliability for several test groups with different running times

If there are several identical products with different running times used in the test (or in the field), each running time completed without failure will contribute to deducing the minimum reliability. Corresponding classifications of the running time are preferable formed for this purpose. Example: The following running times and number of "test specimens" serve as the basis for a confidence level of P_A=80% and a required service life of 100,000 km (assumption b=2):

i	Running time/km	Lv	n
1	100000	1.0	10
2	70000	0.7	20
3	62000	0.62	40

The running times were sorted in descending order and the calculation started at the longest running time. This produces the following points in the diagram:

1) 10 parts survived without failing at the longest running time of L_v=1.0
2) This corresponds to a quantity of 20 parts at L_v=0.7 (identical R_{min}). 20 parts were tested without failure at L_v=0.7
3) Together this results in approx. 40 parts at L_v=0.7
4) This corresponds to a quantity of 50 parts at L_v=0.62 (identical R_{min}). 20 parts were tested without failure at L_v=0.62
5) Together this results in approx. 90 parts at L_v=0.62



The result is a guaranteed minimum reliability of approx. 95%. Referred to the minimum reliability relationship already introduced, the total $R_{min,ges}$ is generally derived from:

$$R_{\min,ges} = 100\% \left(1 - P_A\right)^{\left(\sum_{i=1}^{k} Lv_i^b \, n_i\right)^{-1}}$$

k = Number of different test times (collectiv)

If failures occurred unexpectedly during the tests, the minimum reliability will then be based on the test running time achieved up to this point and the number of test specimens n' still to be tested:

$$R_{\min,ges} = 100\% \left(1 - P_A\right)^{\left(\sum_{i=1}^{k} Lv_i^b + n' \, Lv'^{\,b}\right)^{-1}}$$

k = Number of different test times (collectiv)
n' = Number of test specimens to be tested without failure

Lv' = Test time to be tested for the test specimens without failure

Taking into account previous knowledge

If previous knowledge of the components is available (Bayes method), it can be taken into account by using the Beyer/Lauster method /23/. This previous knowledge can originate, for example, from predecessor models and is expressed by the value R_o that is valid for a confidence level of P_A=63.2%. The expected minimum reliability is:

$$R_{\min} = (1 - P_A)^{\frac{1}{n L_v^b + 1/\ln(1/Ro)}}$$

In the same way as the factor ϕ defined under /26/ for taking into consideration the applicability of the previous knowledge, it is used here under the term previous confidence level to give:

$$R_{\min} = (1 - P_A)^{\frac{1}{n L_v^b + \phi/\ln(1/Ro)}}$$

The previous information factor ϕ must lie between 0…1. ϕ =0 signifies that no previous information should be used whereas ϕ = 1 means all previous information can be used.
ϕ can, for example, assume the following values when the following applies to the components of the earlier tests:

$\phi = 1$	The components and the tests are identical to the current status or are 100% comparable
$\phi = 0.75$	Components have been slightly modified or the design status is identical but from different manufacturers
$\phi = 0.50$	Components have been partially modified, e.g. material properties
$\phi = 0.25$	Components agree only in terms of their concept (rough estimation)

The preliminary confidence level can also be used to express when the test changed. The reduced number of samples is therefore:

$$n = \frac{1}{L_v^b}\left[\frac{\ln(1-P_A)}{\ln(R_{\min})} - \frac{\phi}{\ln(1/R_o)}\right]$$

An acceleration factor can be used to take into account different loads from earlier tests. This acceleration factor is discussed in the following sections dealing with the component strength (service life in the Woehler diagram).
Further details can be found under /23/, /25/ and /26/.

Determining t_{10} (B_{10}) from minimum reliability without failures

If there are no failures in the tests, the following steps can be applied to calculate a t_{10} service life:

Step 1: Determine a minimum reliability R_{min} from existing tests -> Point (1) in the diagram.

Step 2: Determine a mean service life ratio L_{vm}, that is equivalent to the previous tests. The following formula is used for this purpose:

$$R_{\min} = (1 - P_A)^{\left(\sum_{i=1}^{k} Lv_i^b \, n_i\right)^{-1}} = (1 - P_A)^{\left(Lvm^b \, n\right)^{-1}}$$

Rearranging the right side for L_v results in:

$$L_{vm} = \left(\frac{1}{n}\left(\frac{\ln(1-PA)}{\ln(R_{\min})}\right)\right)^{1/b}$$

Step 3: Calculate the reliability value on the Weibull curve with P_A = 50% -> Point (2)

$$R_{PA=50\%} = (1 - 0{,}5)^{1/(L_{vm}^b \, n)}$$

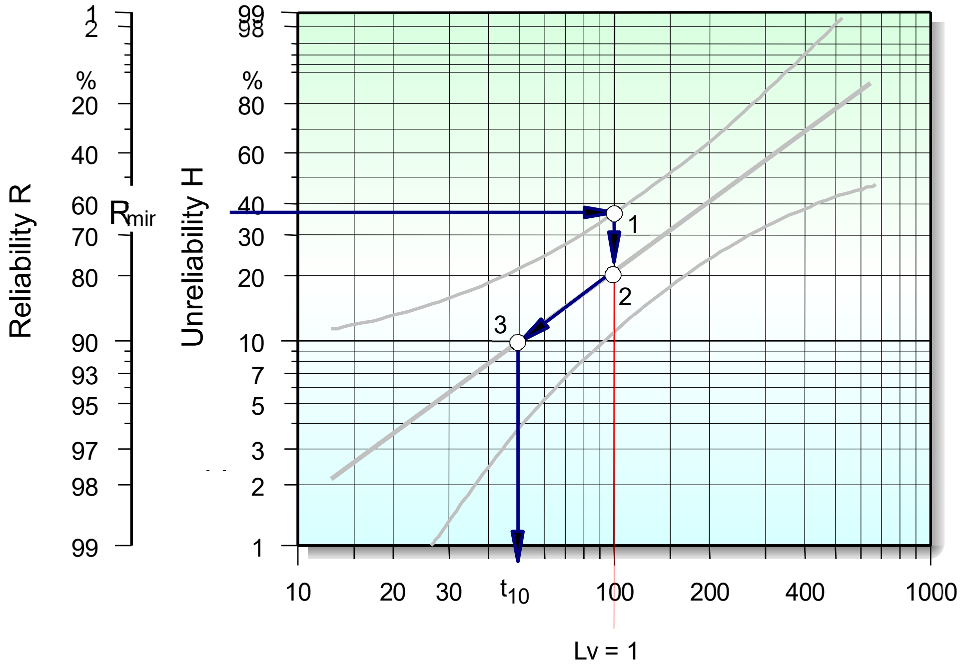
The Weibull curve is now defined by specifying the slope b and the Point (2) on the curve.

Step 4:
Rearranging the Weibull distribution for T results in:

$$T = \frac{t}{(-\ln(1-H))^{1/b}} = \frac{t_{gefordert}}{(-\ln(R_{PA=50\%}))^{1/b}}$$

Step 5:
Calculation of t_{10} (or B_{10}) -> Point (3)

$$t_{10} = T\left(\ln\left(\frac{1}{1-0{,}1}\right)\right)^{1/b}$$



Minimum reliability in tests with unexpected failures

The relationship used to date no longer applies if failures are permitted to occur in the test. R_{min} will then be calculated based on the χ^2 distribution (see /24/):

$$R_{\min} = e^{-\frac{\chi^2_{2r+2,P_A}}{2 \cdot L_v^b \cdot n}}$$

with r = number of failures during the test

Strictly speaking, the previous example with a failure rate at Lv=1.1 is not quite correct for the further calculation. To simplify matters, it was assumed that the test specimen was removed just before reaching the failure at Lv=1.1.

By way of transposition, the necessary new scope of samples is derived from

$$n = -\frac{\chi^2_{2r+2,P_A}}{2 \cdot L_v^b \cdot \ln(R_{\min})}$$

or the necessary testing time

$$L_v = \left(-\frac{\chi^2_{2r+2,P_A}}{2 \cdot n \cdot \ln(R_{\min})} \right)^{1/b}$$

This approach is to be applied when the times at which the failures occur are still uncertain. However, since the number of failures cannot be forecast in advance in practical terms, this calculation is of corresponding significance only for presenting scenarios.

Example: $n=5$, b=2, R_{min}=0.8, P_A=0.9
Necessary testing time for the required minimum reliability

No failure Lv = 1.43
1 failure Lv = 1.87
2 failures Lv = 2.18

Reliability from Binomial-method

In general applies to the statistical assurance without given running time the Binomial-method.

$$P_A = 1 - \sum_{i=0}^{x} \binom{n}{i} (1-R)^i \, R^{n-i}$$

with x = number of failures
n = sample size

R is the the reliability for not defined running time (normally to describe the quality after production). For the representation often the so called "Larson-Nomongram" is used, because the formula can not be resolved for R. Especially in industrial series production the Binomial-method represents an important tool for assessing the quality level for the sampling technique and for the control charts.

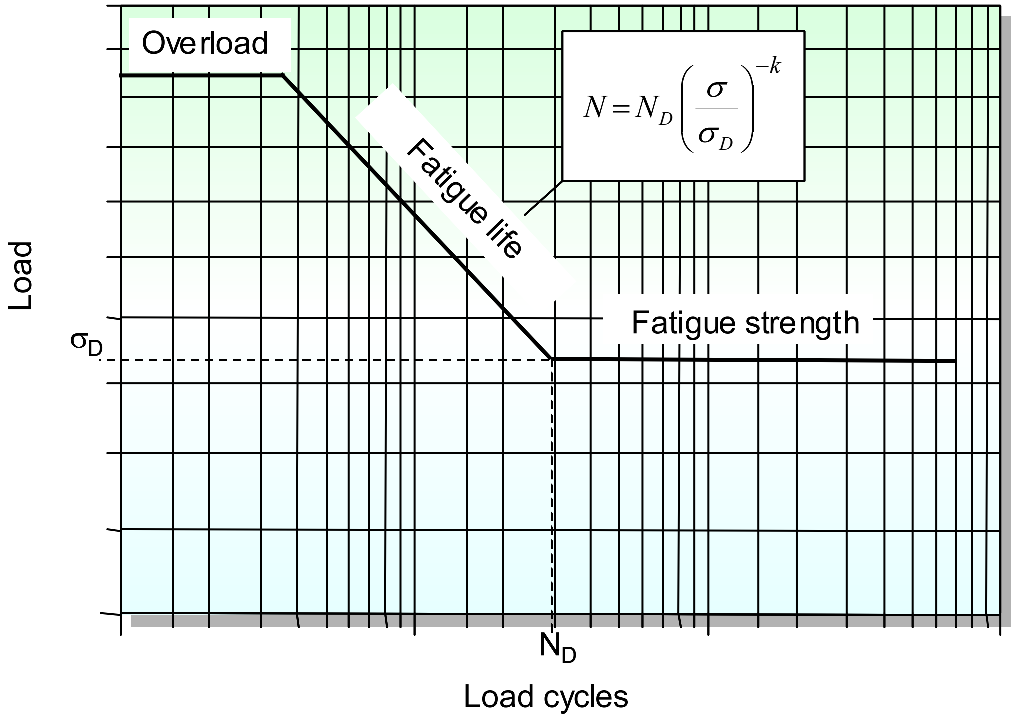
In case of no failures (x = 0) the equation becomes the simple form

$$P_A = 1 - R^n$$

which is conform to the success-run-method.

17. Service life in the stress-cycle (Woehler) diagram

The stress-cycle (Woehler) diagram represents the service life (alternating stress cycles or running time) of a component as a function of load.



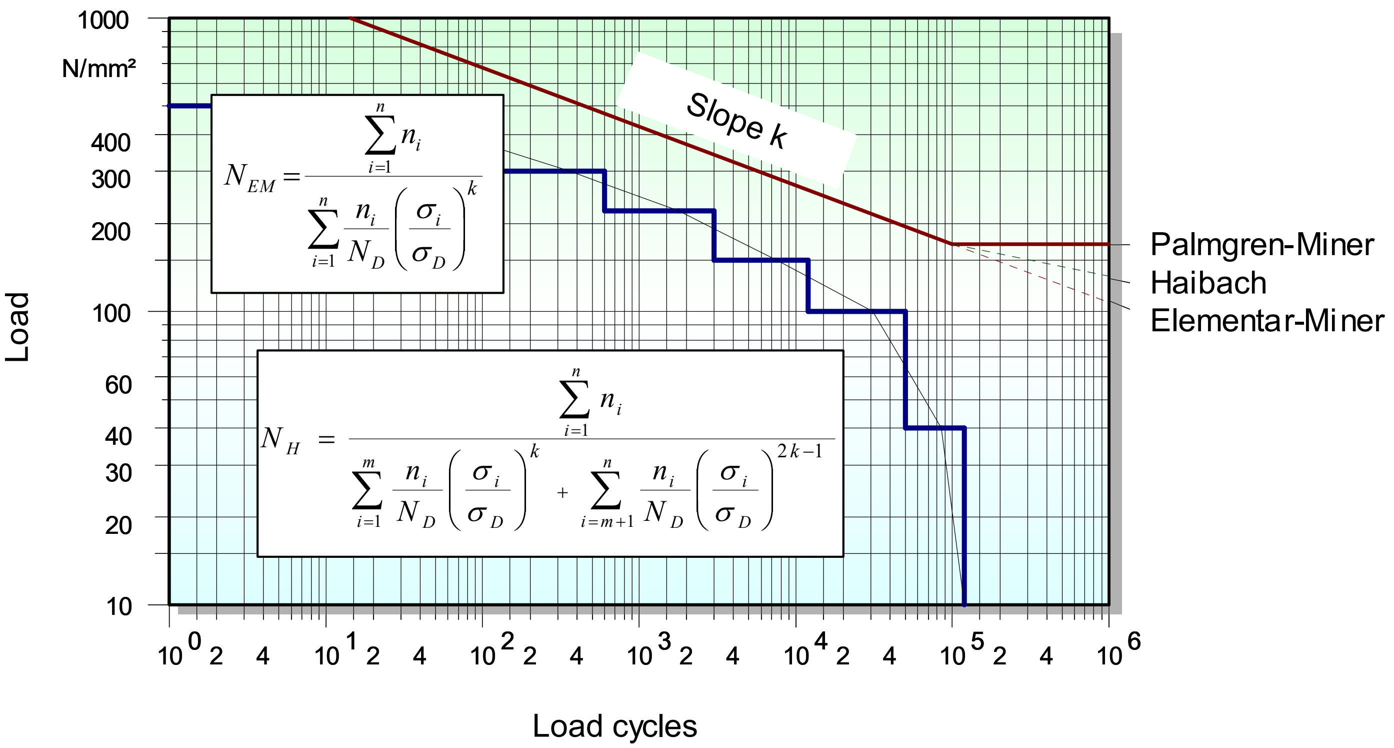
The short-term strength range covers approx. 10^4 alternating load cycles. This figure exceeds a load limit at which damage can occur. In the finite life fatigue strength range a diminishing number of stress cycles occurs until fracture) as the load (component stress) increases. This range is a straight line on the double logarithmic scale. The following Palmgren-Miner relationship applies:

$$N = N_D \left(\frac{\sigma}{\sigma_D}\right)^{-k}$$

where N = Number of stress cycles
N_D = Number of stress cycles as from which fatigue strength exists
σ = Component stress
σ_D = Component stress as from which fatigue strength exists
k = Woehler exponent (e.g. steel k =10..11, cast iron k =13..14, aluminium $k \approx$ 12)

$$\sigma = \sigma_D \left(\frac{N_D}{N}\right)^{\frac{1}{k}}$$

The fatigue strength range begins as from a certain load. As from this point, the service life of the component is no longer dependent on the number of stress cycles. Different loads occur in actual operation that can be determined by way of measurements and grouped in collectives (stages).



The number of stress cycles that can be borne is calculated in accordance with the elementary Miner rule (EM -> Extension of the slope k):

$$N_{EM} = \frac{\sum_{i=1}^{n} n_i}{\sum_{i=1}^{n} \frac{n_i}{N_D} \left(\frac{\sigma_i}{\sigma_D} \right)^k}$$

Past experience has shown that the simple Palmgren-Miner rule defines the service life as too good and the elementary Miner rule as too unfavourable. Haibach therefore proposes to halve the two ranges. Consequently a slope of 2*k-1 is used

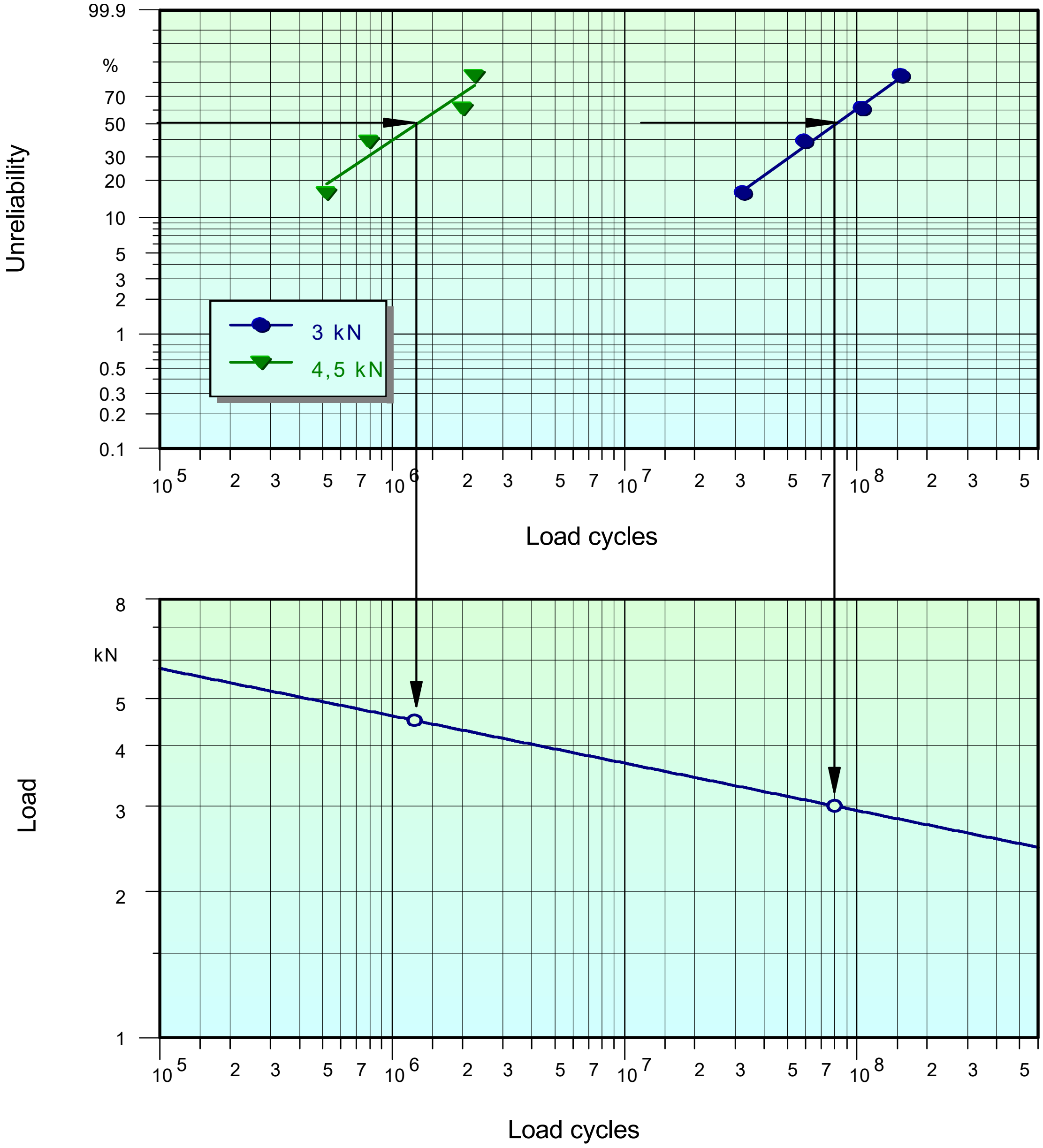
$$N_H = \frac{\sum_{i=1}^{n} n_i}{\sum_{i=1}^{m} \frac{n_i}{N_D} \left(\frac{\sigma_i}{\sigma_D} \right)^k + \sum_{i=m+1}^{n} \frac{n_i}{N_D} \left(\frac{\sigma_i}{\sigma_D} \right)^{2k-1}}$$

and load conditions below the fatigue strength can also be taken into account. The index m refers to the collective up to the fatigue strength kink, i.e. up to σ_D. Past experience has also shown that damage below 0.5*σ_D has no influence.

Deriving stress-cycle Woehler diagram from Weibull evaluation

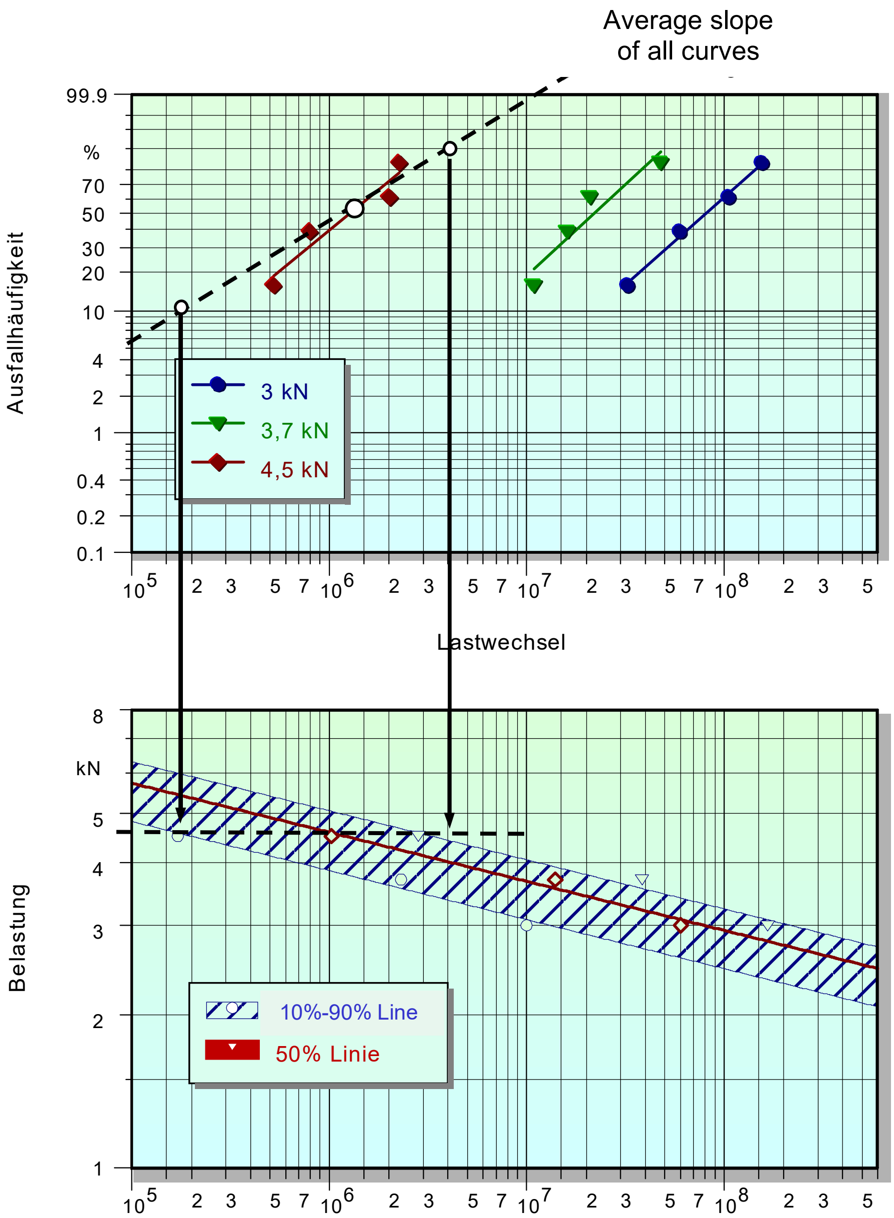
The prerequisite for the failure points within a curve in the Weibull diagram is that they are all subject to the same load. In contract, the stress-cycle (Woehler) diagram, the running time or number of alternating stress cycles is repeated as a function of the component load (ordinates). Consequently, it is possible to draw a conclusion with regard to the expected service life under a certain load. On the other hand, a deduction cannot be made as to what percentage of the components will fail under a certain load.

This is, however, possible through the combination of Weibull evaluation and stress-cycle (Woehler) diagram.



At a certain failure frequency, e.g. 50%, the point in the Weibull diagram is projected downward to the stress-cycle (Woehler) diagram for each load case. The Woehler line can be drawn by connecting these points. In the same way, a probability range can be created in the stress-cycle (Woehler) diagram for a certain range, e.g. 5% and 95% failure probability.

In practical applications, it can be seen that the Weibull gradients or slopes differ under various loads as they are also subject to random scatter. Since the 5% and 95% lines are directly dependent on the slope of the Weibull curves, this results in either expanding or tapering ranges in the stress-cycle (Woehler) diagram. Greater "absolute scatter" of the test results can be expected at higher alternating stress cycles (running times at lower loads).



This comes about, however, not only through an expanding (widening) range but also in a range with a parallel progression due to the logarithmic scale.

In view of the same test conditions, as already described, the slope rates should essentially not differ. It is therefore recommended to use a mean slope b in the stress-cycle (Woehler) diagram to determine the 5% line and 95% line. These lines then run parallel to the 50% line.

This representation is, of course, only possible for the fatigue strength range for finite life. The fatigue endurance strength range, as is typical for steel components and at which the Woehler line changes to a horizontal, cannot be determined as in this case failures no longer occur. Since certain materials more or less always have a certain fatigue strength range at higher running times or alternating stress cycles, when evaluating materials with unknown characters, as many "load points" as possible should be checked in order to determine a kink in the curve.

The slope can be determined by transposing the previous formula for k

$$N = N_D \left(\frac{\sigma}{\sigma_D} \right)^{-k}$$

(Please refer to the section entitled "Service life in the stress-cycle (Woehler) diagram). The second point in the stress-cycle (Woehler) diagram is used instead of the fatigue strength (σ_D) and σ is generally replaced by the force F to give:

$$k = -\frac{\ln\left(\frac{N_1}{N_2}\right)}{\ln\left(\frac{F_1}{F_2}\right)}$$

Woehler with different Loads (Pearl-Cord-Method)

If all tests are carried out with different loads, none can be just provided in the probability chart. The method to project downward from 50% probability is not possible therefore.

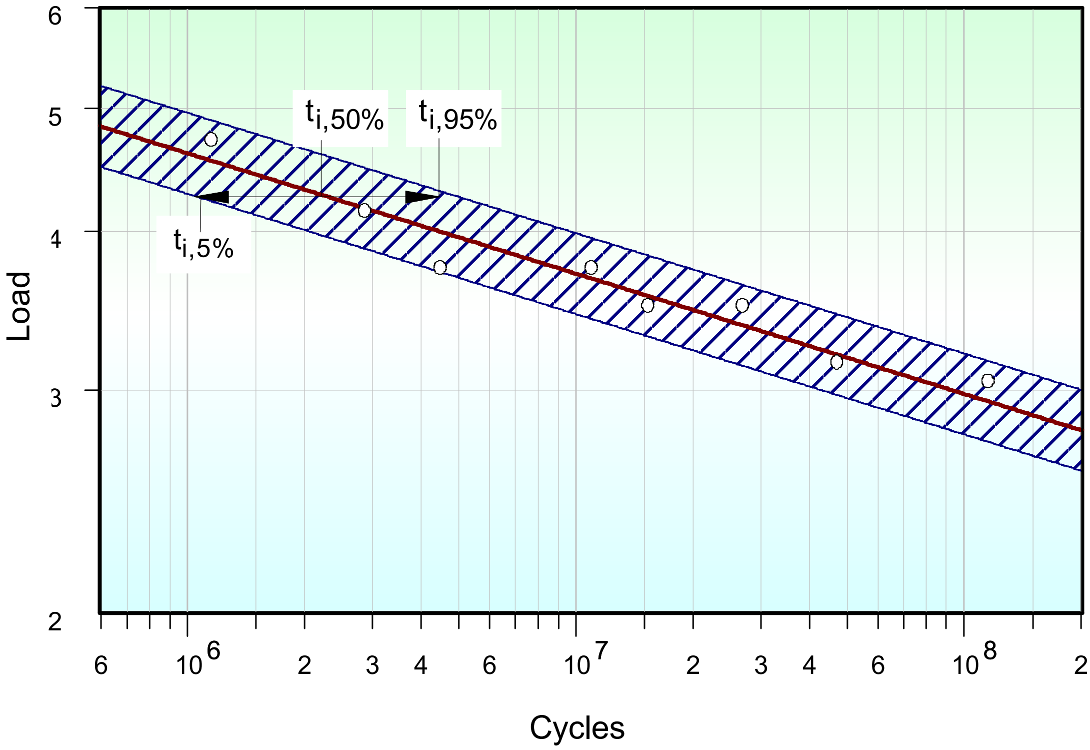
Nevertheless, for the log-normal-distribution there can be used the so called “pearl-cord” method. Condition is that the slope is the same in the probability chart with different loads. That means, we expect that the logarithm standard deviation is constant over the range.

For one load the logarithm standard deviation is:

$$s_{\log} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(\log(t_i) - \log(t_{50\%}) \right)^2}$$

For different laods there are different $t_{50\%}$-values. The logarithm standard deviation can be calculated therefore with:

$$s_{\log} = \sqrt{\frac{1}{n-1} \left(\left(\log(t_1) - \log(t_{50\%,1}) \right)^2 + \left(\log(t_2) - \log(t_{50\%,2}) \right)^2 + ... \right)}$$



The Woehler straight will be now determined through the least square method (points in X- and Y-direction each logartihm).

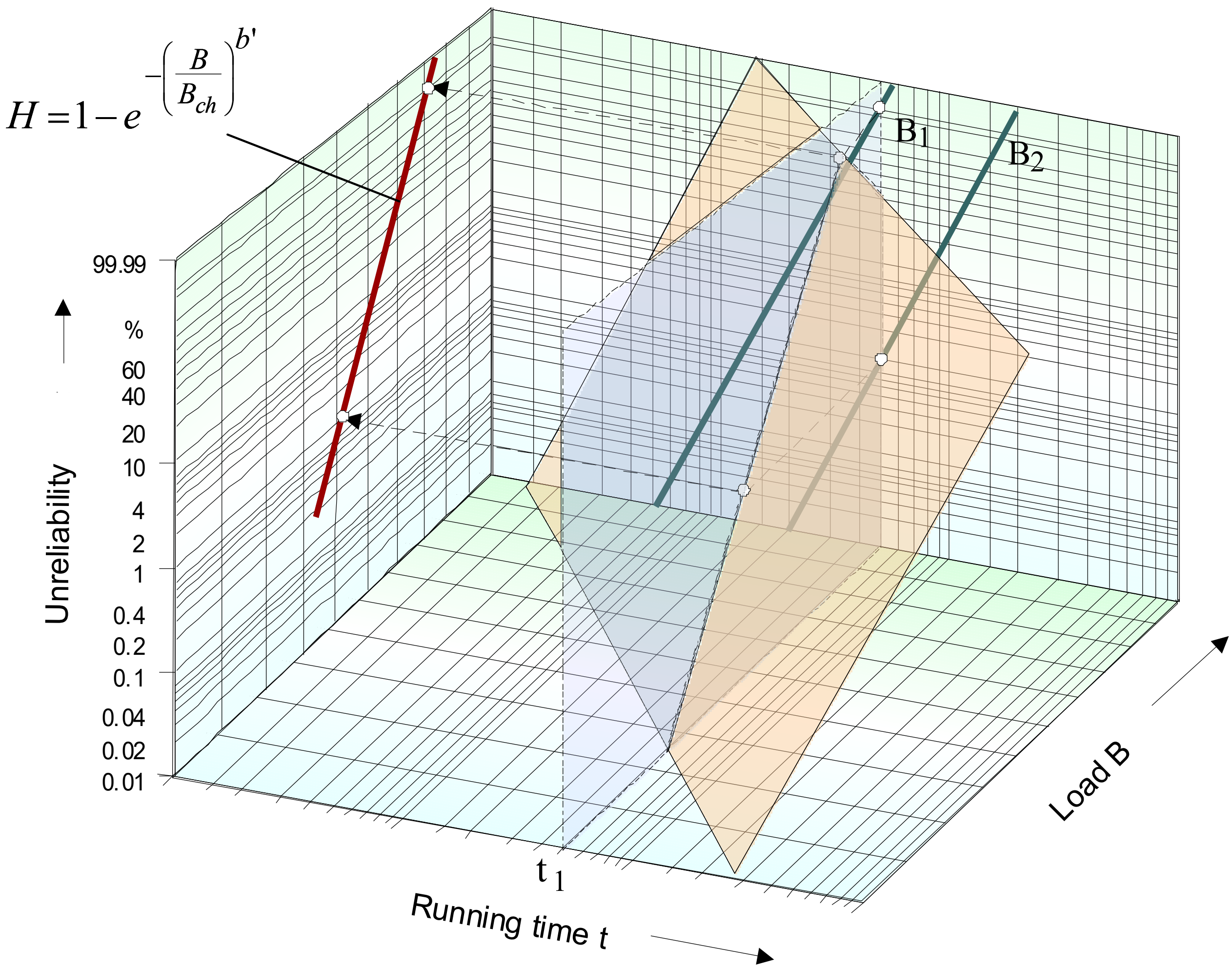
This straight line represents the median- respectively the $t_{50\%}$-values. The probability for 5% and 95% will be determined through the invers-function of the normal-distribution (standardized quantile).

$$t_{5\%} = \frac{x_{50}}{10^{u_{0,05}\, s_{\log}}} \quad \text{and} \quad t_{95\%} = \frac{x_{50}}{10^{u_{0,95}\, s_{\log}}}$$

This method will be used, if there are less test results. But the minimum should be 5 at least. Besides, not all tests must run with different loads. Nevertheless, it should not be used less than 3 load levels.

Weibull plot for different loads

As already mentioned, several times, the X-axis is normally a "running time". A prerequisite for the respective curve in the Weibull plot is that the same load must predominate. However, only the load is possible as the X-axis in the case of tests where the running time is always the same but the load varies. A simple logarithm is taken of the load axis (the Y-axis in Woehler) as well as of the running time axis. The running times now become longer as the load decreases. The following connection can be represented corresponding to the previously defined relationships between Weibull and the stress-cycle (Woehler diagram. 2 loads $B_1 < B_2$ are assumed.



The grey zone represents the relationship between load and failure frequency while the lower level is the stress-cycle (Woehler) diagram.
A defined running time t_1 (cutting plane) applies for transfer to the left-hand diagram, resulting in the red curve for which following relationship applies.

$$H = 1 - e^{-\left(\frac{B}{B_{ch}}\right)^{b^*}}$$

where B_{ch} = Characteristic load analogous to T

The parameters b^* and B_{ch} can be determined using the two points on the green curve (H_1 and H_2).

$$b^* = \frac{\ln\left(-\ln\left(1 - \frac{H_2}{100\%}\right)\right) - \ln\left(-\ln\left(1 - \frac{H_1}{100\%}\right)\right)}{\ln(B_2) - \ln(B_1)}$$

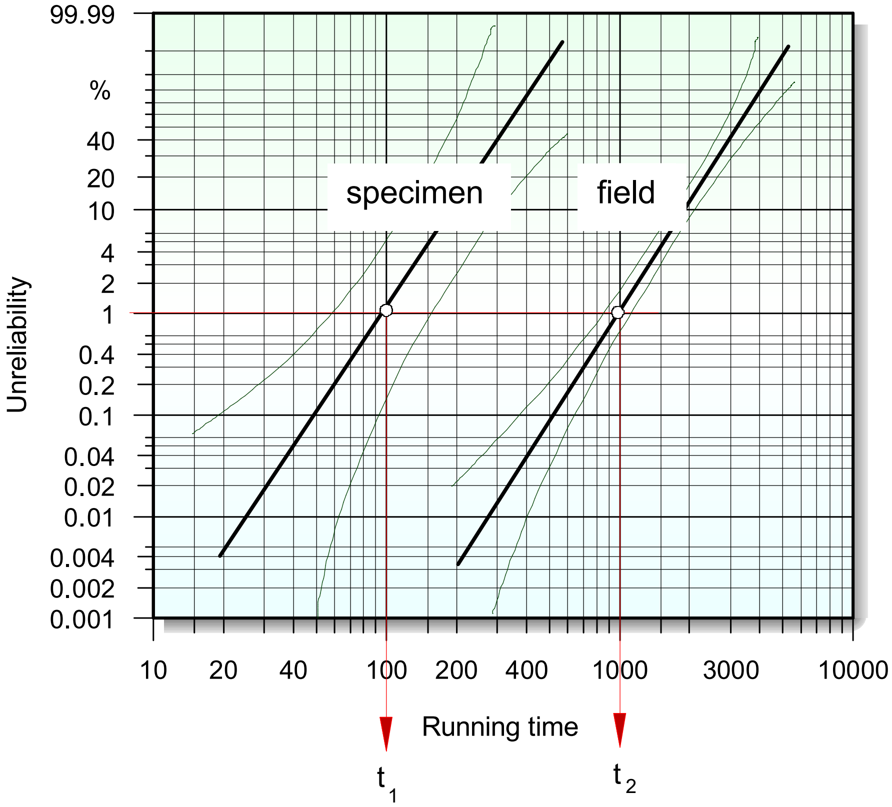
Transposition of the 2-parameter Weibull formula results in the characteristic load represented by:

$$B_{ch} = B_1 \left(-\ln(1 - H_1)\right)^{-1/b^*}$$

The same relationship as in the normal Weibull plot applies. Different curves, to which various running times apply, can also be represented.

18. Accelerated life testing

The aim of accelerated life testing is to shorten the testing time. This is virtually always necessary in the development of components or parts as the required "running times" would be much too long under normal operating load conditions.
Essentially, the test must be designed so as to represent a realistic load case and overload breakage“ does not occur. In the Weibull plot this means that the slopes of the failure straight lines must not differ substantially between test and field.



The accelerated life factor κ is derived through: $\kappa = \frac{t_2}{t_1}$

With a constant slope (parallel progression), κ or the ratio t_2/t_1 is independent of the failure frequency level.
The test exhibits a relatively large confidence bound depending on the number of test specimens. The number of components in the field will generally be considerably higher ($n_{Prüf} << n_{Feld}$), resulting in a smaller confidence bound. If the field observations are complete, the confidence bound will drop to the best-fitting straight line as the entire population is represented.

Case 1: No failures in the test despite increased load

A Weibull plot cannot be produced for tests where no failures occur. Instead corresponding calculations are performed with a minimum reliability in accordance with VDA.

$$R_{\min} = (1-P_A)^{\frac{1}{L_v^b\, n}} \quad \text{or} \quad H_{\max} = 1-(1-P_A)^{\frac{1}{L_v^b\, n}}$$

Lv is the ratio of the testing time to the required service life at constant load as in the field. An accelerated life factor is to be taken into account in the case of a higher load:

$$R_{\min} = (1-P_A)^{\frac{1}{n\left(\kappa L_v\right)^b}} \quad \text{or} \quad H_{\max} = 1-(1-P_A)^{\frac{1}{n\left(\kappa L_v\right)^b}}$$

While the accelerated life factor shortens the actual testing time, it has the positive effect on R_{min} of lengthening the testing time.

Under certain circumstances, it may be necessary to take into account different units for the service life between testing and in-field (e.g. alternating stress cycle and km). An "accelerated life" already prevails if the same alternating stress cycles can be performed within the test faster than during customer operation. This accelerated life“, however, is calculated by correspondingly converting the units. The accelerator life factor κ therefore applies only to the higher load case.

Case 2: Failures occur

The equation for the Weibull distribution is used for the purpose of drawing conclusions with regard to the failures in the field from the failure frequency in testing with the accelerated life factor

$$H_{Feld} = 1 - e^{-\left(\frac{1}{\kappa}\frac{t_{Feld}}{T_{pr}}\right)^{b_{pr}}}$$

included in the calculation. The corresponding "running time" in the exponent can be determined using the inverse function of the distribution resulting in the following form with $T_{field} = \kappa\, T_{pr}$:

$$H_{Feld} = 1 - e^{-\left(\frac{1}{\kappa}\left(\ln\left(\frac{1}{1-H_{pr}}\right)\right)^{\frac{1}{b_{pr}}}\right)^{b_{pr}}}$$

To simplify matters in this case it is assumed that there is no failure-free time t_o.

Determining the accelerated life factor

To determine the accelerated life factor either there must be failures in the field and during testing or it is determined from the load differences in the stress-cycle (Woehler) diagram.
In the stress-cycle (Woehler) diagram, the following applies to only one load in the fatigue strength or finite life range:

$$\kappa = \frac{N_{Feld}}{N_{\Pr}} = \left(\frac{\sigma_{Feld}}{\sigma_{\Pr}}\right)^{-k}$$

However, since there is never a constant load in the field but rather a load collective, the following formula applies to the fatigue strength for finite range or for the elementary-Miner rule (no fatigue endurance strength range):

$$N_{EM,Feld} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\sum_{i=1}^{n} \frac{n_{Feld,i}}{N_D} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^k} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\frac{1}{N_D \sigma_D^k} \sum_{i=1}^{n} n_{Feld,i} \, \sigma_{Feld,i}^k}$$

Generally, only one load is used in testing:

$$N_{\Pr} = N_D \left(\frac{\sigma_{\Pr}}{\sigma_D} \right)^{-k}$$

results in

$$\kappa = \frac{N_{EM,Feld}}{N_{\Pr}} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\frac{1}{N_D \sigma_D^k} \sum_{i=1}^{n} n_{Feld,i} \, \sigma_{Feld,i}^k} \quad \frac{1}{N_D \left(\frac{\sigma_{\Pr}}{\sigma_D} \right)^{-k}}$$

shortened to

$$\kappa_{EM} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\sum_{i=1}^{n} n_{Feld,i} \, \sigma_{Feld,i}^k} \; \sigma_{\Pr}^k \qquad \textit{Elementary-Miner}$$

representing the accelerated life factor for components without fatigue endurance strength. Instead of σ a force or any other variable can represent the load. In accordance with Haibach, the further progression to N_D is assessed with half the slope:

$$N_{H,Feld} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\sum_{i=1}^{m} \frac{n_{Feld,i}}{N_D} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^k + \sum_{i=m+1}^{n} \frac{n_{Feld,i}}{N_D} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^{2k-1}}$$

$$\kappa = \frac{N_{H,Feld}}{N_{\Pr}} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\frac{1}{N_D} \left(\sum_{i=1}^{m} n_{Feld,i} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^k + \sum_{i=m+1}^{n} n_{Feld,i} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^{2k-1} \right)} \quad \frac{1}{N_D \left(\frac{\sigma_{\Pr}}{\sigma_D} \right)^{-k}}$$

Due to the second section with a different exponent, σ_D cannot be cancelled down, resulting in the formula:

$$\kappa_H = \frac{N_{H,Feld}}{N_{\Pr}} = \frac{\sum_{i=1}^{n} n_{Feld,i}}{\sum_{i=1}^{m} n_{Feld,i} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^k + \sum_{i=m+1}^{n} n_{Feld,i} \left(\frac{\sigma_{Feld,i}}{\sigma_D} \right)^{2k-1}} \left(\frac{\sigma_{\Pr}}{\sigma_D} \right)^k \qquad \textit{Haibach}$$

Accelerated life factor for components with fatigue endurance strength range (half taken into account).
The prerequisite for estimating the accelerated life factor from the stress-cycle (Woehler) diagram is accurate as possible knowledge of the material (exponent k) and of the load collective. For literature please refer to /19/ /20/

19. Temperature models

Arrhenius model

In many cases (non-metallic materials), the service life is greatly dependent on temperature. This is particularly true in the case of elastomers and plastics which are used to an ever increasing extent. The Arrhenius model is used for the purpose of representing this relationship. It is based on a chemical reaction with a corresponding reaction rate v. Formula:

$$v = v_o \, e^{-\frac{Ea}{k\,T}}$$

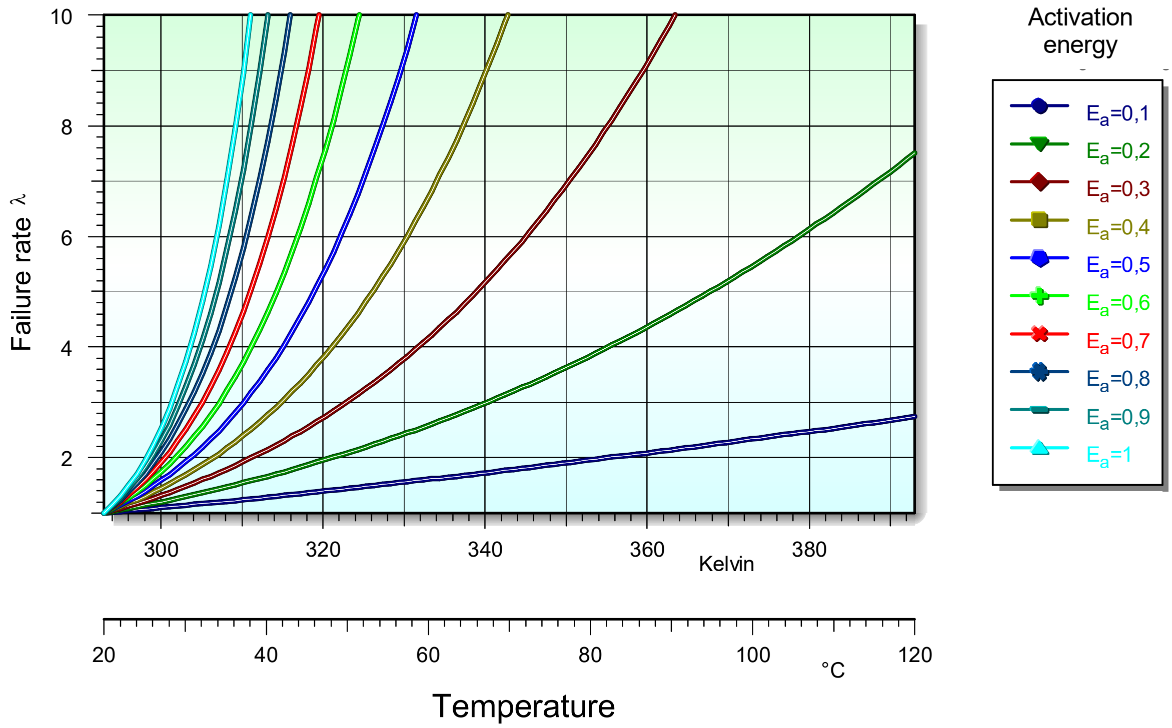
where
v_o : Proportional constant
E_a : Activation energy (component-specific)
K : Boltzmann constant (k=8.617 10^{-5} eV/Kelvin)
T : Absolute temperature in Kelvin

The following applies to failure rates especially for the failure characteristics of electronic components:

$$\lambda_1 = \lambda_o \, e^{-\frac{Ea}{k}\left(\frac{1}{T} - \frac{1}{To}\right)}$$

where λ_o : Failure rate at initial temperature T_o

The activation energy is generally between 0.1 and 1.0 eV.
The following example shows the scaled failure rate (for λ_o=1) in the range from 20 °C – 120 °C.



The failure rates increase at higher temperatures ($T_1 > T_o$) and a so-called acceleration factor can be defined:

$$A = e^{-\frac{Ea}{k}\left(\frac{1}{T1} - \frac{1}{To}\right)}$$

Among other things, this relationship can be used for accelerating procedures in laboratory tests. The acceleration factor therefore has the same effect as the accelerated life factor κ at a constant initial failure rate over time (b=1).

The prerequisite is the knowledge of the component-specific activation energy which, if not known, must be determined by way of tests. The rule of thumb that an increase in temperature by 10 °C doubles the failure rate or halves the service life is often used as the basis for calculations in the range between 70 °C and 120 °C.

The disadvantage of the Arrhenius model is that the failure rate is used and not the lifetime itself.

Coffin-Manson model

The following formula is used in cases where different loads apply:

$$N_2 = N_1 \left(\frac{B_1}{B_2}\right)^k$$

Coffin-Manson used this relationship with k = 2 for the temperature dependency:

$$N_2 = N_1 \left(\frac{\Delta T_1}{\Delta T_2}\right)^k$$

ΔT : Temperature change

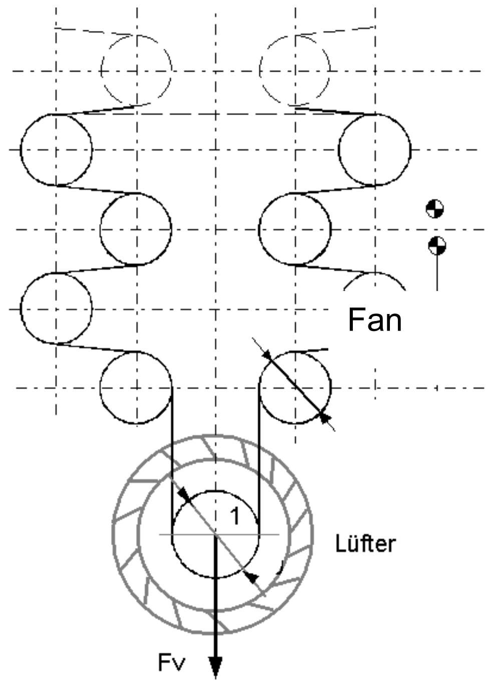
k : Material characteristic

This therefore makes it possible to use Woehler fundamentals and methods. An offset can also be used instead of ΔT ((T_1-$Offs$)/(T_2-$Offs$)). The parameter k and the $Offs$ must be determined by testing.

20. Highly Accelerated Life Tests

HALT Highly Accelerated Life Test

The highly accelerated life test (HALT) serves the early detection of development and process weaknesses (is the production process suitable?) The load is increased in stages at a stress level far above that of normal operation (combination of mechanical stress + temperature increase). This test is mainly used for electrical and electronic assemblies. A variant of this test for mechanical systems is the *Failure Mode Verification Test.* The figure to the right shows a heat reverse bending cycle test rig for ribbed V-belts.



- ⇨ Suitable for detecting early failures
- ⇨ Disadvantage: Weibull parameters cannot be determined, permissible limit?

HASS Highly Accelerated Stress Screening

The highly accelerated stress screening test (HASS) serves the purpose of securing and increasing series production quality -> 100% of production is screened. In contrast to the HALT test, a reduced load is used. The components must not be damaged -> a suitable stress level must be determined. The HASS test generally detects changes in processes and production.

HASA Highly Accelerated Stress Audit

The highly accelerated stress audit HASA is used for testing random samples.
This procedure involves destructive testing of the component which is then no longer used.

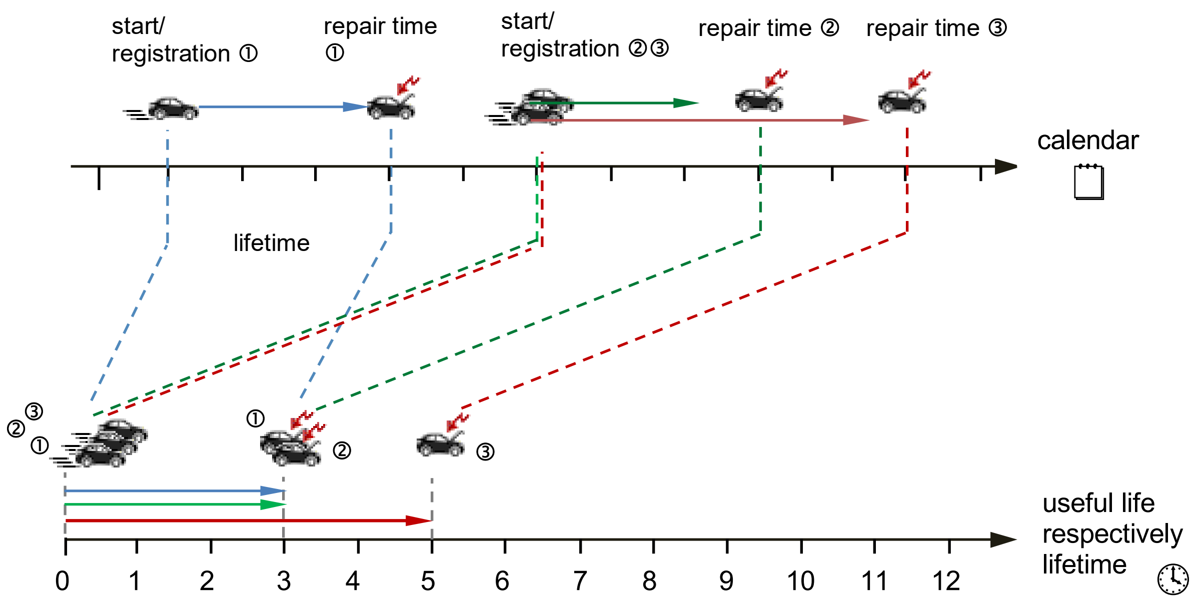
21. Field prognosis

Data preparation for field analysis

If there is service life data in which not all parts have failed, this data is called incomplete or censored. In particular, if parts that are still intact have shorter service lives than others with a failure, the failure frequencies must be corrected. This description is about data preparation.

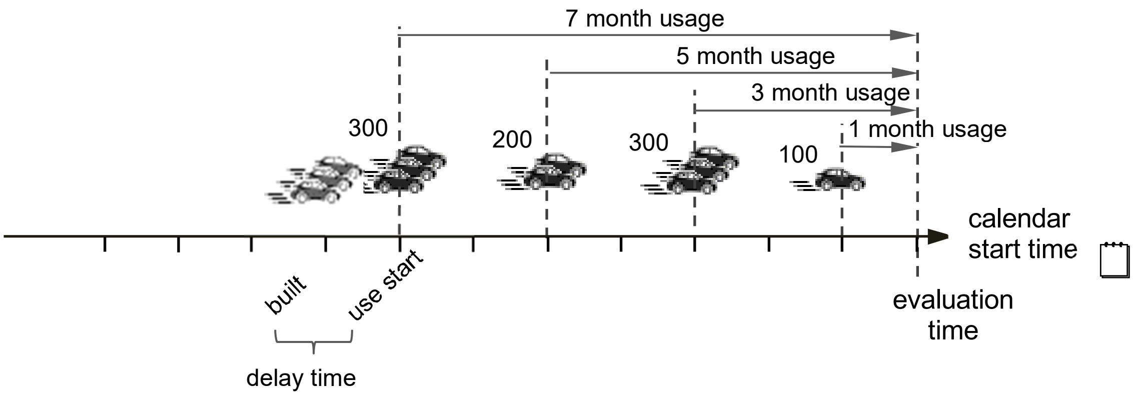
This description shows how to properly split the data from the failed and healthy units. This is necessary, for example, in order to be able to use the Johnson or maximum likelihood method, which is widely used for censored data. Furthermore, it is about several possible errors or parts in a system.

How is the service life and useful life of the intact parts determined? The useful life until failure or the service life is always the time between the start of usage and the time of failure. When considering the useful life (lower graphic), the calendar starting point is irrelevant.



The useful life shown in the lower time axis corresponds to the x-axis in the Weibull diagram i.e., the service life.

Now how are the intact (censored) units divided and their useful life determined, which they will still live to see? To do this, we look at the period from the start of use to the time of evaluation:



With the number of units produced in the calendar period, it must be taken into account that these units will be used later or will start later. This time is referred to as the delay time. As a rule, an average delay of 1-2 months is assumed. If the month of production and the registration date are known e.g., for vehicles, the average delay time can be calculated from this. This results in the table of failures and the table of units produced:

complaints

month	number
3	2
5	1

production

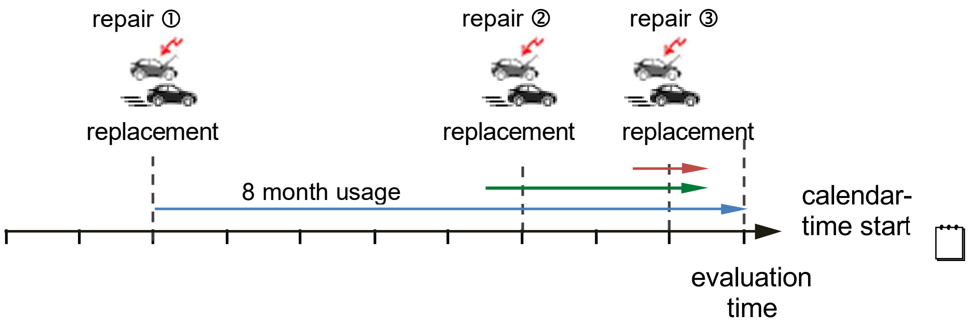
month	number
1	100
3	300
5	200
7	300

For the representation in the Weibull diagram, failures and the intact units must be listed with the times they have been in use. The intact units result from the production quantity minus the failures. Physically, the concrete failures must be subtracted from the quantity where they were produced. In simplified terms, allocation is often made based on the usage time, which results in the following table:

Intact units

month	number
1	100
3	298
5	199
7	300

First of all, the number of failures plus the number of intact units has to add up to the total production. Since every failure has to be replaced by a spare part, the reference value n increases again by the number of failures. The previous deduction of the failures from production cannot be compensated for here, because the useful life of the failures and the spare parts are different:



In addition, there are 3 further units with 8, 3 and 1 months useful life after installation up to the time of evaluation:

replacements intact units total number intact

month	number
1	1
3	1
8	1

+

month	number
1	100
3	298
5	199
7	300

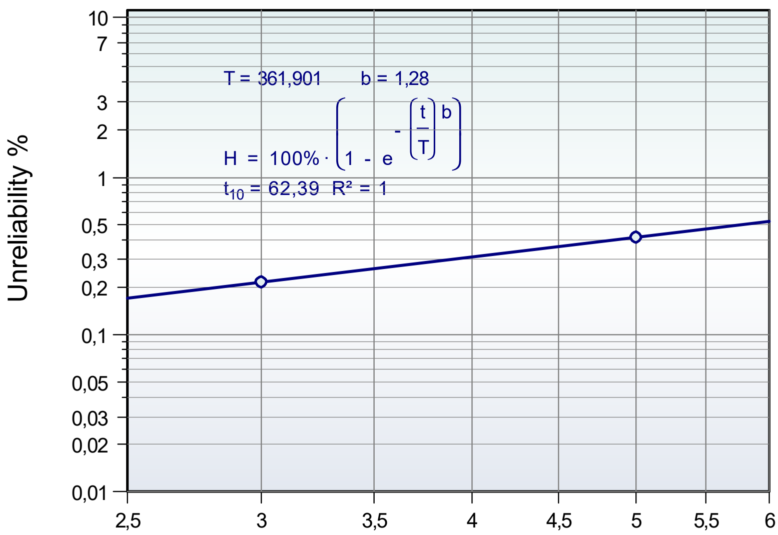
⇨

month	number
1	100+1
3	298+1
5	199
7	300
8	1

Ultimately, the Weibull distribution table results in the following combined table of failures and total number of intact units. The intact units are marked here by definition with a minus:

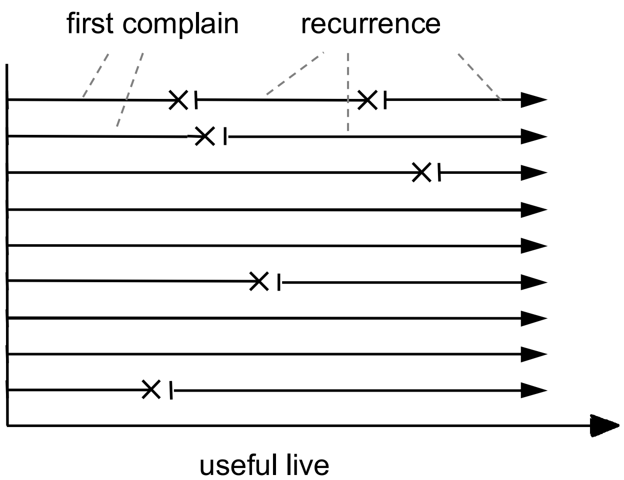
month	number
1	-101
3	2
3	-299
5	1
5	-199
7	-300
8	-1

This table gives the Weibull net shown on the right with the Johnson method.



Repeated failure of a repaired part

In the event of a complaint, the affected part will be replaced with a new one, as described. However, this replacement (spare part) can fail again. When that happens, it's called a recurrence event. The useful life of the repeat case must relate to the time of the previous failure.



The logic is the same as for the spare parts described in the previous section.

In principle, one can say that all spare parts that are still "running" increase the number of units by defining their useful life in a table. Any parts that failed from the original production reduce the number of intact units from that production.

Several different components in the system

For the reference variable n for calculating the median rank values, the number of system units (e.g. vehicles) must be multiplied by the number of parts, because theoretically a system can often fail due to its parts. Mathematically, this assumes that the parts fail independently of each other.
Correctly, one would have to make a separate Weibull analysis for each component, because each component has its own failure characteristics and thus different b. The system is then combined into a common Weibull via a serial block diagram. In other words, the overall reliability of the system is known $R_{system} = R_{comp1} \cdot R_{comp2} \cdot R_{comp3} \; .... \; R_{compz.}$ However, the problem then is that some parts have too few failures to be able to determine a representative component reliability using a component Weibull. For this reason and for reasons of simplification, a "system Weibull" should be made here. However, the determined gradient b of the system can only be interpreted to a very limited extent or not at all, because it is a mixture of different causes of failure. However, this is more about a business perspective than a correct Weibull interpretation.

Several identical components in the system

A good example of identical components are the spark plugs in the engine. A 4 cylinder has 4 spark plugs. The reference variable n would therefore have to be multiplied by 4 in relation to the number of vehicles. However, the spark plugs cannot be individually identified or distinguished. In addition, they are always exchanged together in customer service. They should therefore be considered as a unit.
The situation is different with redundant components. If there are two identical processors that make a controller redundant for safety reasons, these are to be differentiated with processor 1 and processor 2, for example. The failure characteristics would even be the same here. Otherwise, the same considerations apply as in the previous chapter.

A redundancy increases the availability of the system, but not the situation of the repair (an exception are silent redundancies where the first failure is not noticed).

Simultaneously exchanged parts per complaint

In the event of a complaint, several parts are often exchanged at the same time.
There are different scenarios here:

1. All parts are independently defective at the same time
2. he failure of one part causes the failure of the others in whole or in part
3. Only one part is really defective, the others have been exchanged without justification

Case 1 is very unlikely, case 2 is more realistic. Only a component analysis can clarify this, but this is often not possible for practical reasons. The parts would have to be returned for this. However, if an analysis is possible, the unauthorized exchanged parts should be removed from the list of complaints.

If this is not done and a Weibull evaluation is nevertheless created, this is more of a business point of view, because the costs and spare parts have been incurred. However, as already mentioned, the gradient b determined can only be interpreted to a very limited extent or not at all.

Tabular representation

1. Single failure

Ident	Prod. date	Start/regist	Repair date	Part/variant
BDP58219	14.5.2021	15.6.2021	21.2.2022	Powerunit

Ident occurs only once
⇨ Failure with 8 months of use (21.2.2022-15.6.2021)
⇨ Intact service life spare part added (eval. date - 21.2.2022)

2. Case of recurrence

Ident	Prod. date	Start/regist	Repair date	Part/variant
BDP52353	15.3.2021	22.4.2021	30.6.2021	Change aggregate
BDP52353	15.3.2021	22.4.2021	30.4.2022	Change aggregate

Same ident, production, approval and same part
⇨ First failure with 2 months of use (30.6.2021-22.4.2021)
⇨ Repeat case 10 months usage (30.4.2022 - 30.6.2021)
⇨ Intact service life spare part added (eval. date - 30.4.2022)

3. Multiple replacement

Ident	Prod. date	Start/regist	Repair date	Part/variant
BDN51721	16.4.2021	13.5.2021	27.6.2021	Change aggregate
BDN51721	16.4.2021	13.5.2021	27.6.2021	joint top

Same ident, production, registration, repair but different parts
⇨ 2 Complaints with 1 month usage (27.6.21-13.5.2021) (early failure)
⇨ 2 x intact service life spare parts added (eval. date - 27.6.2021)

As described, it must apply here that all parts were defective at the same time, independently of one another, which is a deliberate simplification.

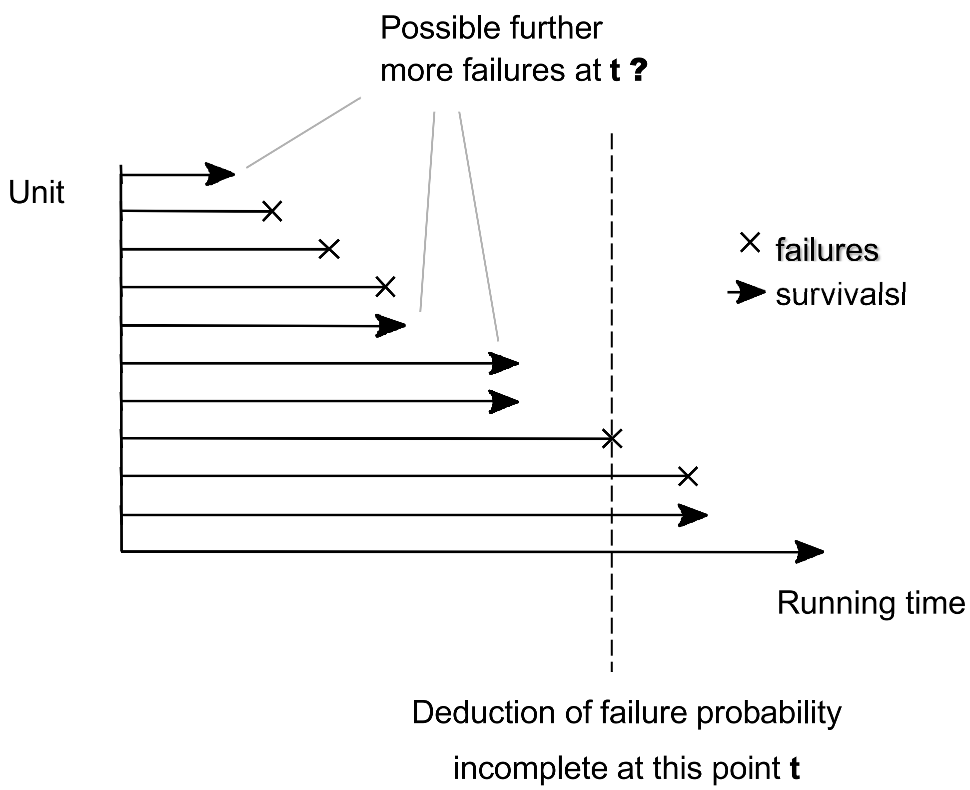
Weibull evaluation for operating hours or km

If it is not the calendar time that determines the service life, but rather the actual useful life or the distance traveled in km, then information about the operating hours or the distance traveled is also required. A system may have mechanical and electronic components that can fail. For example, a vehicle transmission also has a control unit. The number of operating hours is decisive for this, while the km distance is more relevant for the mechanical scope. If the system is evaluated as a whole, you have to decide on a unit. It is recommended to consider the critical component here and to relate the unit to it.
The calculation for operating hours or km is based on the same principles as described, except that the service life shown in the Weibull network is no longer the difference between repairs and the start of use, but the specified hours or km. The question, however, is how the intact units are scored. The answer is relatively simple if you have an average number of operating hours/month or mileage/month. With this, each period of use can be converted into months in the alternative "lifetime unit".
If the parts produced still have an expected lifetime of 10 months up to the evaluation date and the average mileage/month is 2000 km/month, then 20000km is entered in the table for these parts. The mean mileage distribution information can be estimated from the outage data by dividing the km by the usage time.

It should be noted that the determination of the running distance distribution from the complaint data is a one-sided random sample analysis. It is also said that this is a negative selection that only refers to failed units and is not representative of the whole. Therefore, if possible, the distribution of running distances should be obtained from other data sources that have a larger sample of at least 1000 units. The same applies to the distribution of operating hours.

Candidate field prognosis for mileage (automotive)

Generally, relatively large deviations occur in the analysis of failures in the field based on long service lives. The cause is as follows: For example, when observing the frequency of certain fault cases on the Weibull plot at a distance (mileage) X, a certain proportion of the production quantity has not yet covered this distance and can therefore also not have failed.



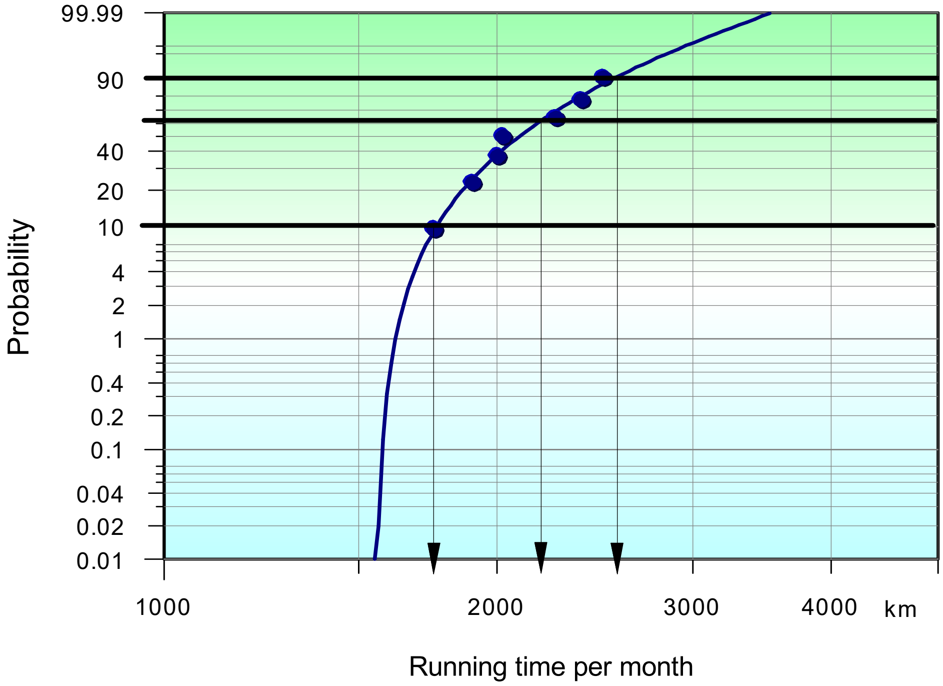
A correct conclusion as to the frequency of the failures at a distance (mileage) X can be drawn only when all produced components have also reached the corresponding mileage X. The analysis becomes progressively better the greater the time between the actual analysis and manufacture of the components. This is simply due to the fact that it is more likely that all components have reached a defined service life or mileage. It is, however, also necessary to draw the earliest possible conclusion with regard to "field data". This therefore means it is necessary to devise a method of including a prognosis of the components that have not yet failed. These components are known as "candidates“. A procedure for determining these candidates is described in the following (please refer to /3/).

It is assumed that the candidates have the same failure probability as that of the failed components. If the statistical kilometre reading (mileage) of the vehicles under observation is known (mileage distribution), the failure probability of the candidates can be concluded by way of relatively simple calculation.

The distance or mileage distribution may be ascertained both from the fault cases where the date of registration, date of fault and distance covered (mileage) are known as well as by conducting corresponding enquiries. The distance or mileage distribution indicates the percentage of vehicles that has not reached a defined service life (mileage). The distribution is appropriately scaled to mileage/months and can therefore be adapted in linear form to any other time scale. In practical application, however, it is necessary to take into account that this distance or mileage distribution is not constant. For example, the mileage will be low during the running-in period of a vehicle whereas the mileage will then increase year on year. It is also necessary to take into consideration the fact that the same components in different types of vehicles and under different operating conditions but also in different countries will achieve greatly varying mileages (e.g. taxis have very high mileage readings). The distance or mileage distribution is normally a lognormal distribution. However, since this is not available as a function, the Weibull distribution is used either with t_o or it is set out over 2 sections. For example:

Distance or mileage distribution after 1 month
X1 10.0% 720 km
X2 63.2% 2160 km
X3 90.0% 3700 km

At the first reference point X1", 10% of the vehicles have not yet reached 720 km. At X2, 63.2% of the vehicles have not yet reached 2160 km and so on. After a month (observation period), this distance or mileage distribution appears as follows in the Weibull plot:



From this representation it is possible to determine how many vehicles have not yet reached a specific mileage. From a mathematical point of view, the inverse Weibull function is necessary at this point (see annex). It is therefore possible to calculate the number of candidates from the production quantity. Let us assume the following concrete failures of a certain component are analysed, sorted in ascending order of distance covered (mileage):

Ordinal i	Failure times at km
1	2000
2	2800
3	4500
4	5000
5	6000
6	7000
7	8000
8	8000
9	9000
10	10000
11	11000
12	13000
13	15000
14	17000
15	20000

The corresponding candidates are initially determined as a percentage for each distance covered, at which a failure occurred, represented in the distance or mileage distribution diagram. The absolute number is now calculated based on the production figures. Initially, this quantity applies to the first value (candidate at 2000 km). The following candidates are now calculated from the subsequent number of vehicles that have not yet covered the corresponding distance (mileage), minus the previous candidates and failures. The calculation may result in a negative number of candidates that must be set to 0.
A "corrected“ failure frequency can now be determined from the candidates. This is calculated using:

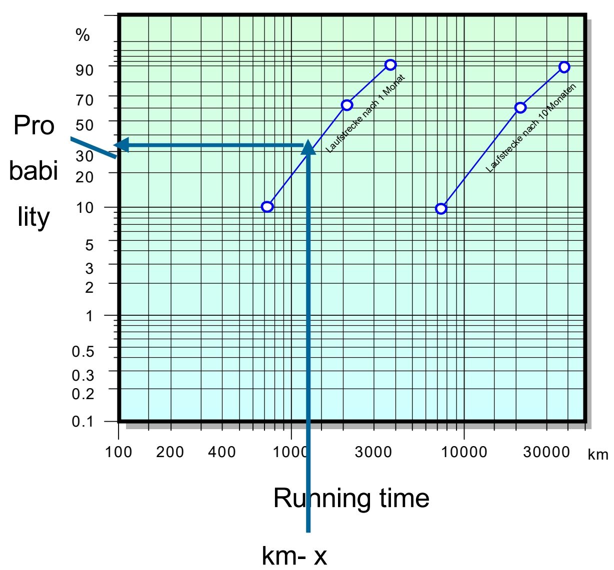
$$Fan_i = Fan_{i-1} + \frac{1}{n+1} \cdot \frac{Nan_i}{1-Fk_{i-1}}$$

* n

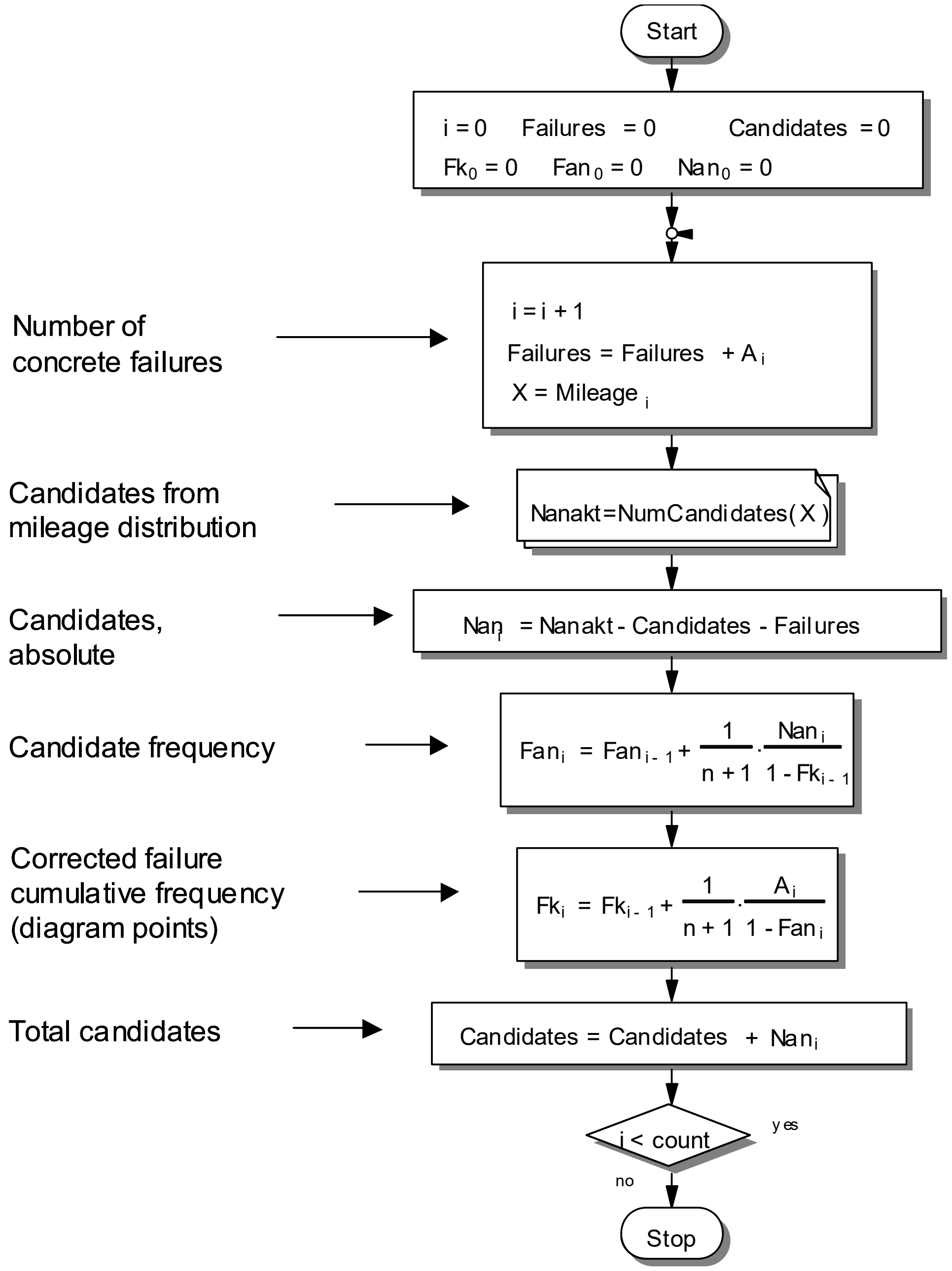
with the candidate frequency

$$Fk_i = Fk_{i-1} + \frac{1}{n+1} \cdot \frac{A_i}{1-Fan_i}$$

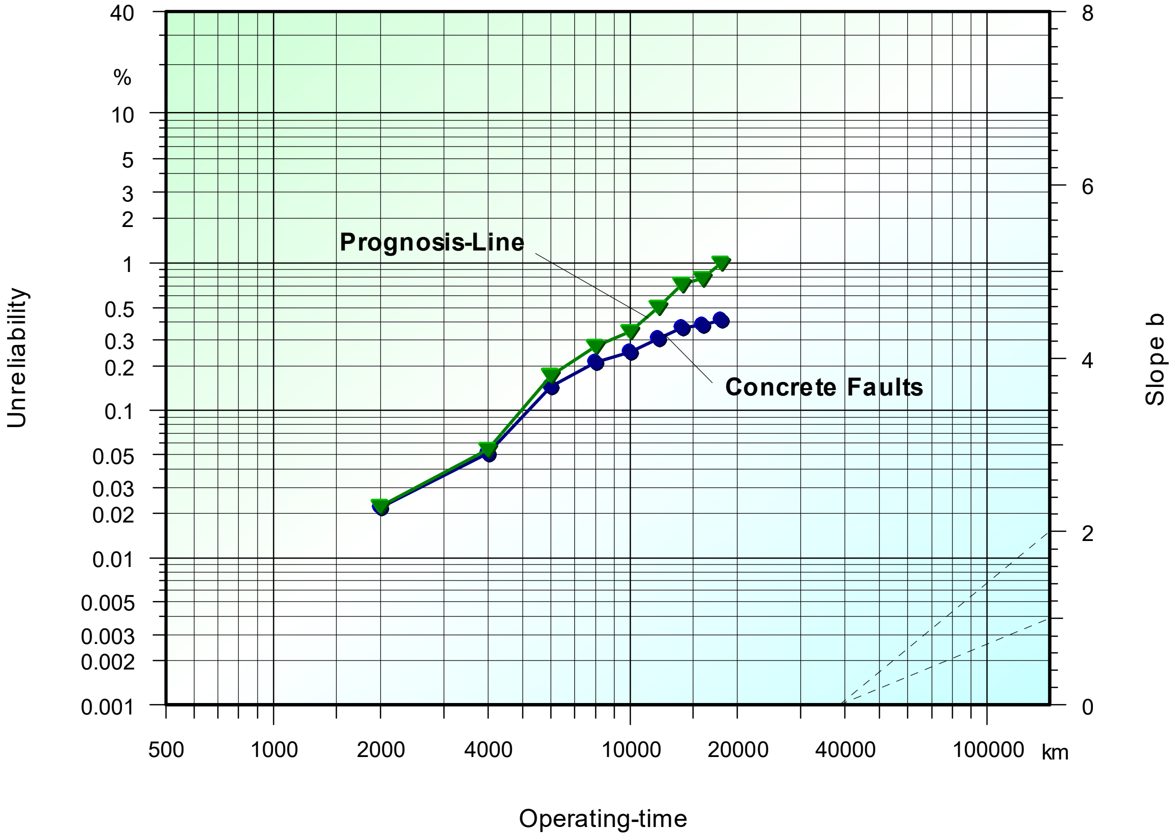
where i : Ordinal
n : Production quantity
A_i : Number of failures at point i
Nan : Number of candidates from-distance distribution



The following diagram shows the entire calculation procedure in simplified form:

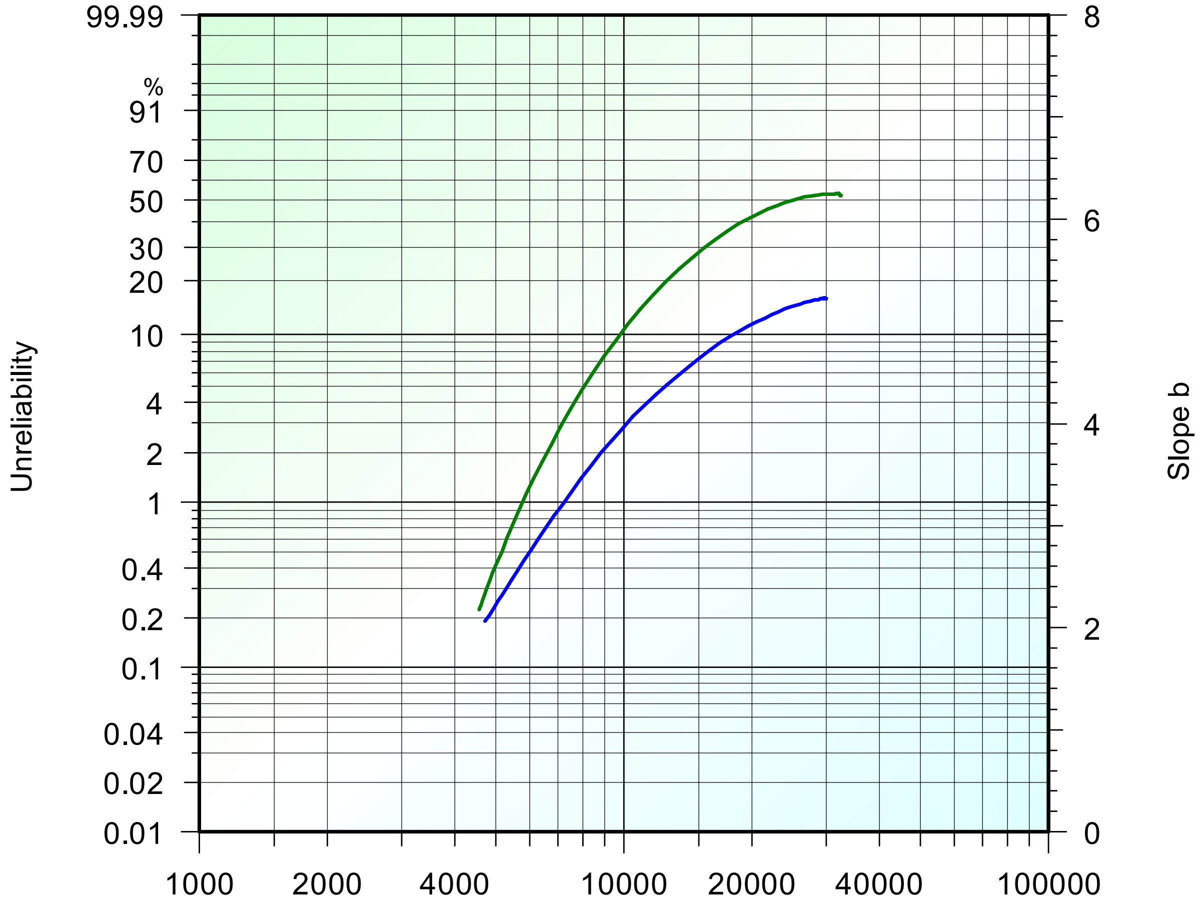


The initial values for starting the calculation for i=0 are set to 0.
In the case of the above example, the following corrected failure frequencies or prognosis line are achieved based on a production quantity of 999:



It is particularly noticeable that the corrected failure frequencies become effective only after the distance covered (mileage) has become longer as less and less vehicles have already reached the corresponding distance/mileage (large number of candidates).
The implementation period which is equivalent to the observation period has a particular influence on the corrected failure frequency. As already mentioned, the line of the corrected failure frequencies will always be closer to the line of the concrete failures the further the observation period lies in the past or if it is very long. In this case, all vehicles have already reached the corresponding mileage and there are no new candidates. In addition, component analysis should be performed only in a very closely delimited period of time (max. 3 months) as the observation period applies only to one "point" (mean value between the first component and the component last produced).
In the literature it is recommended to classify the data in case of more than 50 failure points, but the disadvantage is to detect mixed-distributions and unregularities. Note: not all differences to a straight line have technical reasons.

A kinked progression (shallower start) often occurs over the distance covered, also for the prognosis or forecast line.



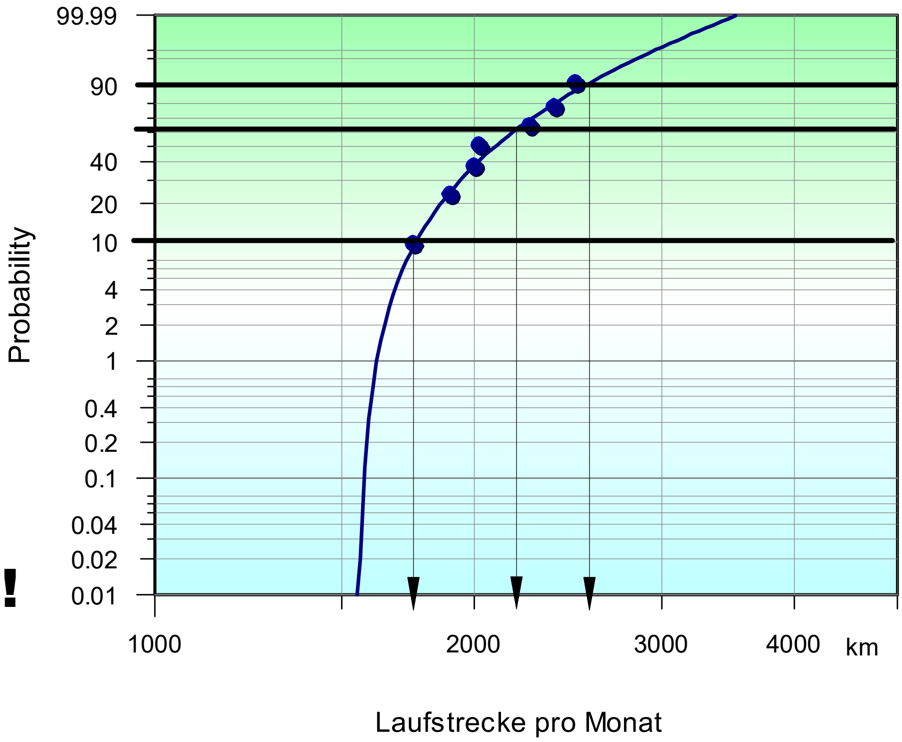
This may be attributed to the fact that the defects occur outside the warranty period and are therefore no longer included here. The data are considered as elapsed although they involve concrete "failures". As a rule, this conclusion can be drawn when the "kink" occurs above 40000 km. In this case, it is very probable that the warranty period has elapsed. If the kink is below 40000 km, it must be assumed that only a certain "assembly" or a limited production batch is affected and thus, despite the higher kilometre reading (mileage), no further failures are to be expected.
For further information please also refer to the sections "General evaluation problems" and "Mixed distribution".

Determining distance or mileage distribution from "defective parts"

As described in the introduction, the distance or mileage distribution can also be determined from the defective parts data. For this purpose, the following data are required for each defective part:

- Vehicle registration
- Repair date
- Kilometre reading (mileage)

The "operating time" of the component is determined from the difference between the repair date and vehicle registration. These data are entered in a table together with the distance covered (mileage). As mileage distribution can be determined only for the same operating periods, the data are scaled to one month, i.e. the mileage is divided by the time in operation.

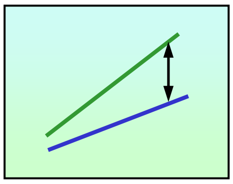


Running time per month

The point of intersection of the best-fitting line obtained from these data at 10%, 63.2% and 90% frequency (probability) provides the required kilometre values at these points. Perpendiculars are drawn at the corresponding frequencies for this purpose (see example). The determined values can now be adopted directly in the actual Weibull evaluation.

! It must be borne in mind that this evaluation is based on a "negative selection". This means that, in most cases, the vehicles with the failures have a higher m ... sentative cross section. Using the distance or mileage distribution based on defective parts therefore normally means obtaining a relatively low prognosis.

Candidate prognosis/characteristics



- ⇨ The deviation between the prognosis (forecast) and actual curve is greater the shorter the observation period.

- ⇨ The delay (time between production and registration) shortens the observation period accordingly.

- ⇨ The deviation between the prognosis and actual curve is greater the lower the distance covered (mileage) per month.
  -> The probability for candidates that the observed distance was not reached increases.

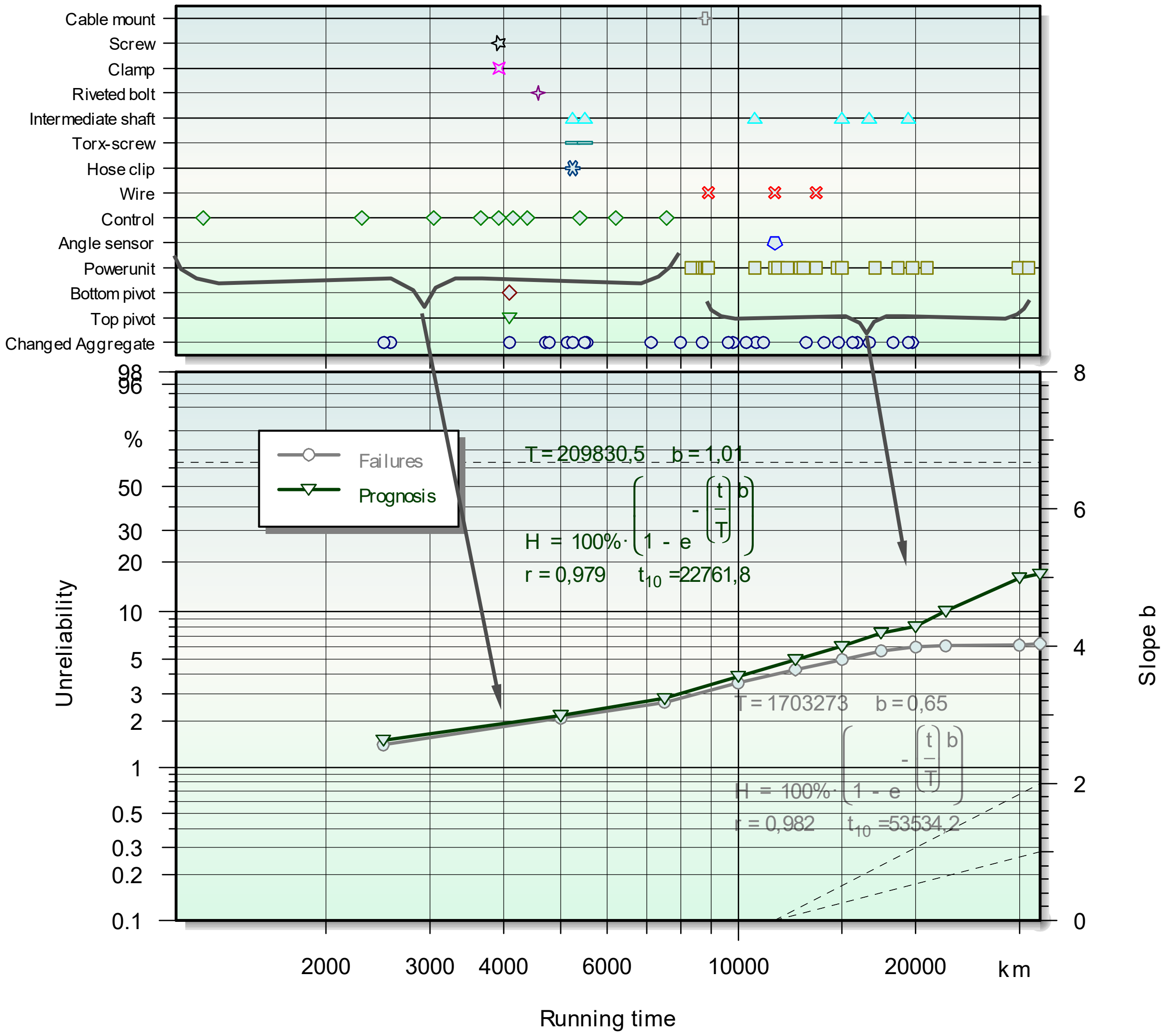
- ⇨ Mileage distributions based on defects is often too high and not always representative for overall production.

More detailed analysis with parts evaluation

Different components are generally replaced in connection with Weibull analyses based on problems established by way of defect or fault codes. If this information is available together with an identification number (ID), additional parts evaluation can be performed that may be of assistance in finding the cause of problems.

	A	B	C	D	E
1	**ID**	**Registration**	**Rep. date**	**Mileage**	**Part**
2	BDN51721	28.5.2004	4.10.2004	4095	Changed Aggregate
3	BDN51721	28.5.2004	4.10.2004	4095	Top pivot
4	BDN51721	28.5.2004	4.10.2004	4095	Bottom pivot
5	BDN53182	28.6.2004	13.9.2004	4147	Powerunit
6	BDR90410	2.4.2004	23.11.2004	10742	Changed Aggregate
7	BDS63617	5.8.2004	13.10.2004	2574	Changed Aggregate
8	BDP58219	18.5.2004	25.2.2005	14727	Powerunit
9	BDR29919	30.3.2004	6.8.2004	2511	Changed Aggregate
10	BDS63432	1.3.2004	28.2.2005	13975	Changed Aggregate
11	BDR30136	13.4.2004	30.4.2004	1218	Powerunit

Only the vehicles (or products) appear classified in the Weibull plot. In the majority of cases, however, several parts are replaced within one ID and therefore one problem case. These parts can be represented in terms of distance (mileage) by the Weibull diagram.



More points than in the classified Weibull diagram, which also shows only the number of vehicles (products), appear in the above diagram. The important factor in this diagram is the respective assignment of the kilometre ranges.
This example exhibits a range of random failures (up to approx. 10000 km) for the candidate prognosis curve as well as a wear-related section in the upper area. The distributions of the *transmission* and *power unit* parts are significant in this diagram. It is possible to conclude that, based on the kilometre assignment, the *transmission* is more likely to fail due to production faults (slope $b \approx 1$) while the *power unit* in the upper range of the slope at $b \approx 2$ is conspicuous due to wear.

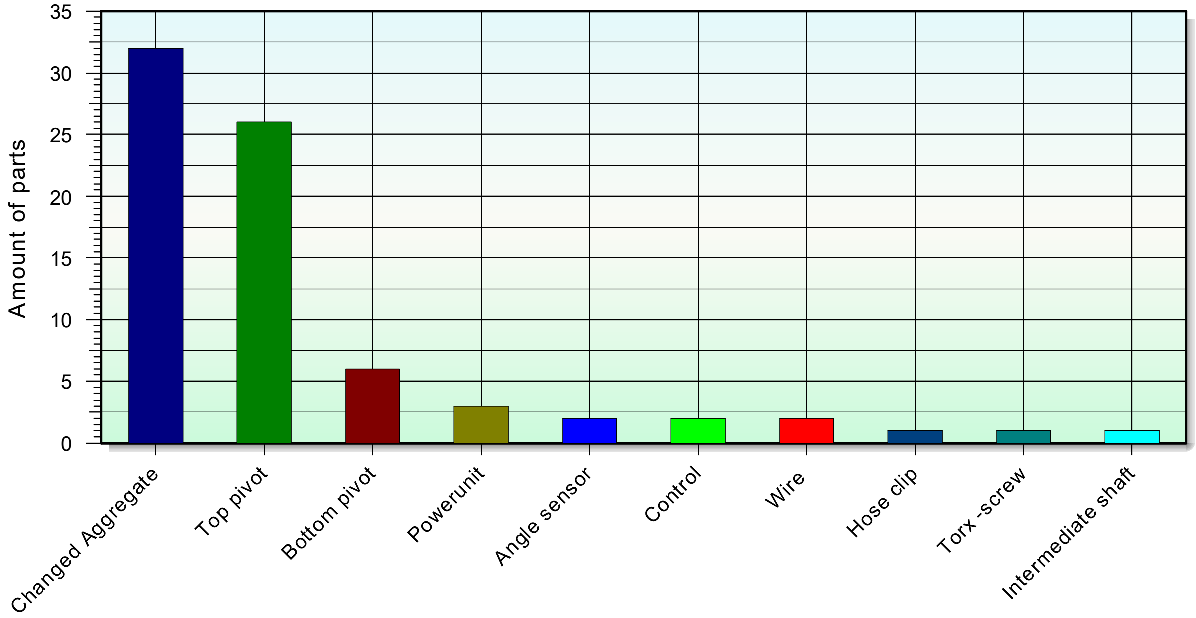
No distinct conclusion can be drawn for other parts. In practical applications there is the added problem that service technicians do not always make the correct diagnosis and parts are often replaced that are actually in working order. The analysis is therefore correspondingly fuzzy depending on the circumstances.

The option of achieving improved parts analysis would therefore include, for example:

⇨ Subsequent technical parts analysis

⇨ Plausibility check

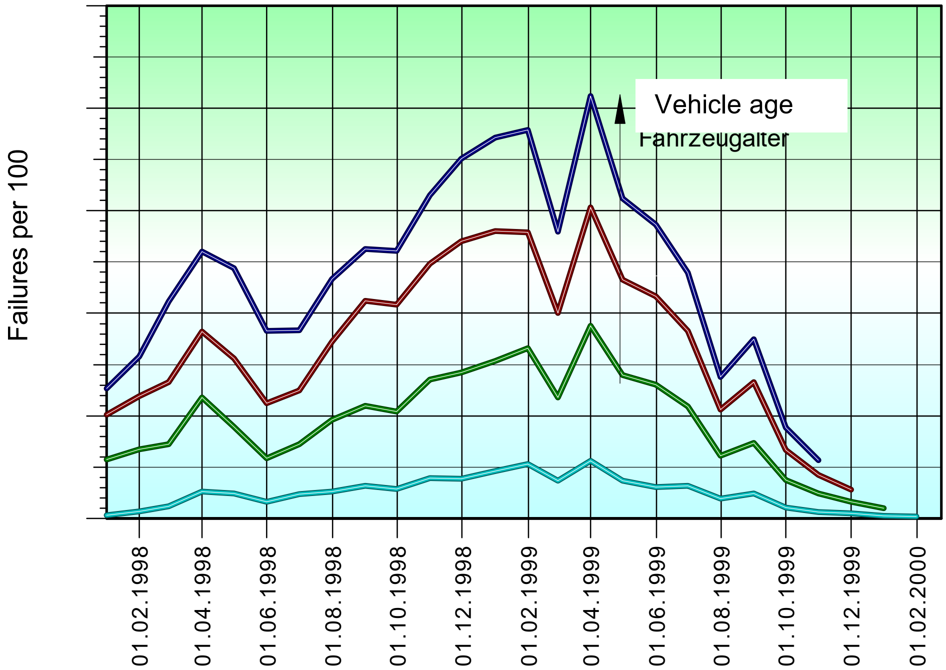
Ultimately, only a Pareto distribution can be derived from the number of parts in order to indicate where the focal points of defects are located.



A further point to be borne in mind relating to the above problems is that when replacing certain parts, inevitably other parts must also be replaced, especially small parts such as screws, cable straps etc. Other cases involve seals that must always be renewed when replacing the actual defective parts. These elements are, of course, of no significance with regard to the analysis as it only concerns the important components such as the power unit or transmission in our example.

22. Contour-plots “Schichtlinien”

The problems that occur over the production month are collected in so-called contours . Contour lines are plotted representing constant vehicle age. This type of diagram is therefore also known as an isochrone representation. When considering the line with the vehicle age of 6 months, for example, all current problems relating to vehicles that are 6 months old are entered 6 months back from the observed period (production month). All vehicles aged 12 months are entered in the next line 12 months back. This results in a "historic“ record of the problems over time.



As a rule, the produced parts differ from month to month with regard to their tolerances and properties as well as in the assembly process, indicated by substantial fluctuations in the line progressions.

Advantages of contours

- ⇨ Overview of changes in production quality and production process
- ⇨ Assignment of corrective action possible in series production

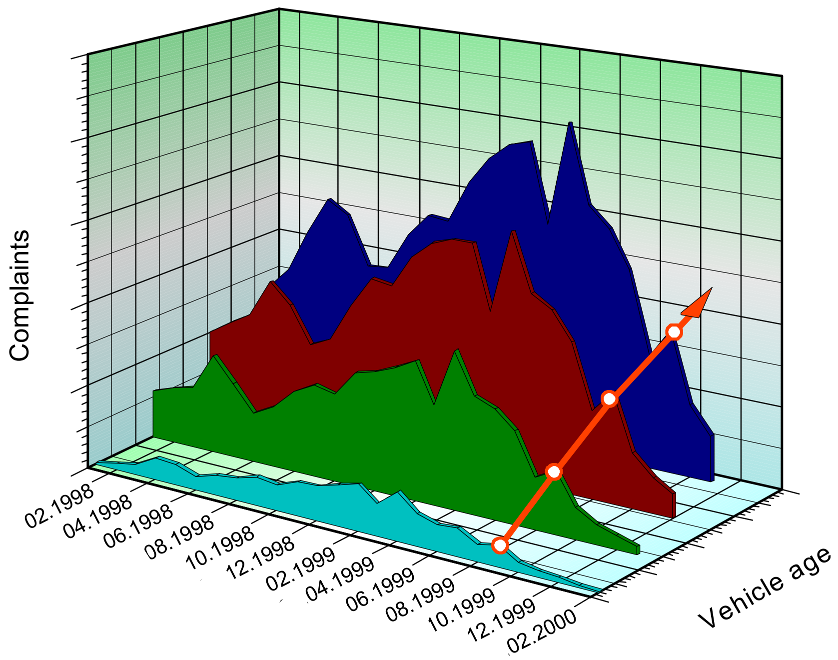
Disadvantages

- ⇨ The causes of the problems are not indicated
- ⇨ Lines with vehicle age lying far in the past break down with respect to time so that continuation is possible only as a prognosis
- ⇨ Percentage of repeat problems not recognisable

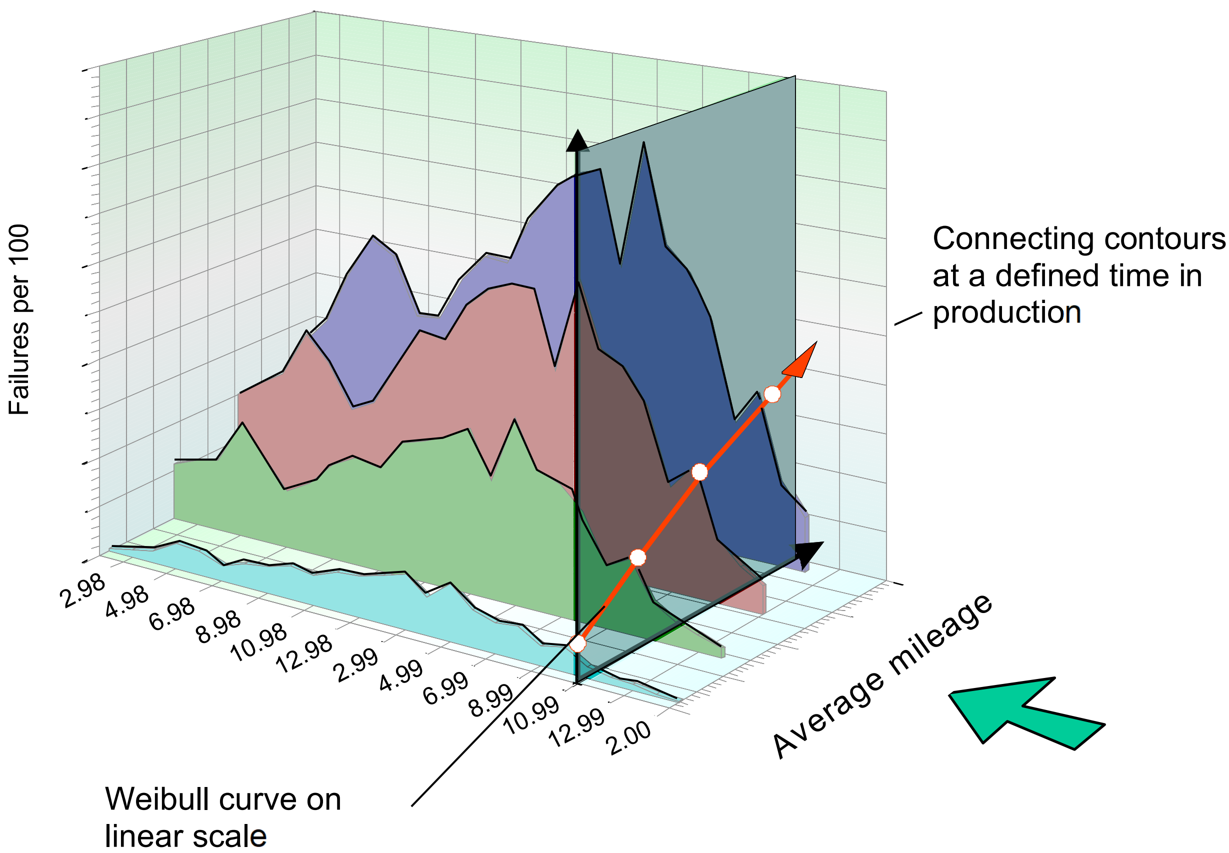
In the majority of cases, performing Weibull analysis for each month means involved data processing and is therefore correspondingly time-consuming.
However, it is possible to estimate the Weibull parameters quickly and simply directly from the contours.

Weibull parameter b from contours

When the contours are considered as a 3D representation, the vehicle age is indicated in depth. With the mileage of the vehicles known and, to simplify matters, assuming constant mileage over the vehicle age, this axis also corresponds to the distance covered.



Viewing the diagram from the right-hand side, it can be seen that the representation is nothing else than a Weibull diagram with linear axes.



Corresponding to the number of contours, points (red) are obt ained for a certain production month. These points can be used to determine the Weibull parameters in pairs (two points are required for the two-parameter form). By rearranging the Weibull functions for T

$$H_1 = 100\% \left(1 - e^{-\left(\frac{t1}{T}\right)^b}\right) \qquad H_2 = 100\% \left(1 - e^{-\left(\frac{t2}{T}\right)^b}\right)$$

and equating followed by subsequent resolution for b, the following result is obtained for the two points (t_1,H_1) and (t_2,H_2):

$$b = \frac{\ln\left(-\ln\left(1 - \frac{H_2}{100\%}\right)\right) - \ln\left(-\ln\left(1 - \frac{H_1}{100\%}\right)\right)}{\ln(t_2) - \ln(t_1)}$$

The kilometre values (mileage) that are derived from the vehicle age times the mean distance covered per month ($t = A * LproM$) are obviously required in the denominator. However, since the logarithms t-values are deducted from each other, it is not necessary to convert the actual kilometre values. It is simply sufficient to specify the months as per the following:

$$\ln(A_2 \cdot LproM) - \ln(A_1 \cdot LproM) = \ln(A_2) - \ln(A_1)$$

$$e^{\ln(A_2 \cdot LproM) - \ln(A_1 \cdot LproM)} = e^{\ln(A_2) - \ln(A_1)}$$

$$\frac{A_2 \cdot \cancel{LproM}}{A_1 \cdot \cancel{LproM}} = \frac{A_2}{A_1}$$

With A = vehicle age in months it is therefore possible to simply determine:

$$b = \frac{\ln\left(-\ln\left(1 - \frac{H_2}{100\%}\right)\right) - \ln\left(-\ln\left(1 - \frac{H_1}{100\%}\right)\right)}{\ln(A_2) - \ln(A_1)}$$

The driving distance or mileage distribution is therefore not necessary for determining b alone. The information derived from the contour is sufficient for this purpose. However, the actual operating time is to be used for A_1 und A_2. The delay that lies between production and actual vehicle operation is to be deducted. If influence is not negligible and must be at least estimated if no concrete data are available.
The transposition of the two-parameter Weibull formula results in the characteristic life T:

$$T = e^{\frac{b \ln(t_1) - \ln\left(-\ln\left(1 - \frac{H_1}{100\%}\right)\right)}{b}}$$

The kilometre value (distance) of t is, however, required for this formula. Instead of the characteristic life T in km, a characteristic vehicle life T^* in months can also be used, resulting in:

$$T = T^* \cdot LproM$$

Entered in the 2-parameter Weibull equation this results in:

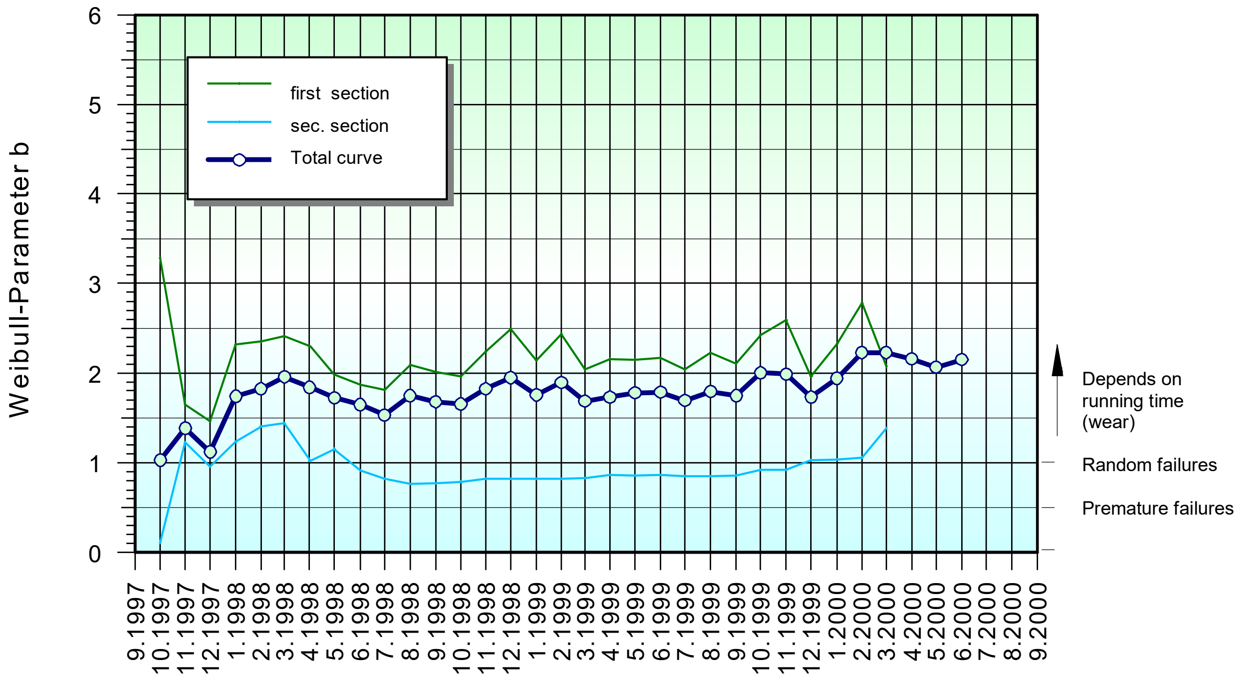
$$H = 100\% \left(1 - e^{-\left(\frac{t}{T}\right)^b}\right) = 100\% \left(1 - e^{-\left(\frac{A \cdot LproM}{T^* \cdot LproM}\right)^b}\right) = 100\% \left(1 - e^{-\left(\frac{A}{T^*}\right)^b}\right)$$

and rearranging for T^* :

$$T^* = e^{\frac{b \ln(A_1) - \ln\left(-\ln\left(1 - \frac{H_1}{100\%}\right)\right)}{b}}$$

The delay must also be taken into account in this case. Generally, only the shape parameter b is of interest for the purpose of distinguishing between early failures, random failures and wear failures.

Complete b and T corresponding to the standard Weibull plot, can be determined by constructing a Weibull straight line from the points of the contour for each production months.
A so-called "**b-Chart**" is now obtained by plotting the calculated b over the production period, providing an overview of the "production process".



The following additional information can be derived from the above diagram:
The thick line in the middle shows the respective b of the overall Weibull progression of each production month. The thin curves represent two sections of the Weibull progression. The light green curve is the first section and the light blue represents the second section in the Weibull plot.
If the light blue curve for the second section is now located below that for the first section, this indicates that the Weibull curve flattens off in upward direction and kinks to the right. Under certain circumstances this situation may indicate faulty batches. If the first section is below the second, the Weibull curve kinks to the left, i.e. it is steeper towards the rear. This is probably attributed to a mixed distribution that requires further investigation. Different failure mechanisms can then be expected.

The aim is to achieve the largest possible b and the lowest possible failure rate (corresponding to a steep straight line located far to the right in the Weibull plot). b<=1 indicates the presence of chance and early failures, the "process" is not in order. A large difference between the smallest and largest b within a production month indicates a substantial curve in the Weibull plot. In such cases, it is advisable to produce the Weibull diagram for this period, which is easily possible following the described procedure.
The Weibull plot can be produced directly based on the relative problems as "failure frequencies" and the respective vehicle age multiplied by the mean distance covered (mileage). From the mileage distribution, however, it is known that for each vehicle age 50% of the vehicles have not yet reached the mean distance (mileage) at the time the data are taken. Problems can therefore be expected in connection with these vehicles with the same failure probability. In the same way as the prognosis calculation, this therefore results in "candidates". They are derived from the production quantity for the observed production month minus the vehicles already with problems multiplied by 50%.

$$H_{Anw} = 100\% \left(\frac{n - n \cdot H / 100\%}{n} \right) \cdot \frac{50\%}{100\%} \cdot H \quad = \quad \left(100\% - H\right) \cdot 0{,}5 \cdot H / 100\%$$

The forecast total failure frequency for the respective point is then:

$$H_{ges} = H + H_{Anw} \quad = \; H + \left(100\% - H\right) \cdot 0{,}5 \cdot H / 100\%$$

$$H_{ges} \approx 1{,}5 \cdot H$$

It should be noted that the data of the contours may contain double findings (repeated vehicle problems) that are undesirable in the Weibull evaluation. In addition, the fact that vehicles with a certain age do not all cover the same distance per month is also neglected (mileage distribution). Nevertheless, the Weibull plot observes the failure points referred to a specific kilometre value, resulting in a king of classification. If the classic Weibull evaluation is performed and compared with the described procedure, it will be seen that the actual failure points are far apart on the horizontal. This results in corresponding deviations in the comparison of the respective b-values. Strictly speaking, the candidates should also be taken into consideration when determining the individual b-values. However, since the candidates shift the failure level for all points upward by approximately the same relative frequencies, the b-values will change only insignificantly. For simplification reasons, the candidate calculation should therefore not be taken into account.

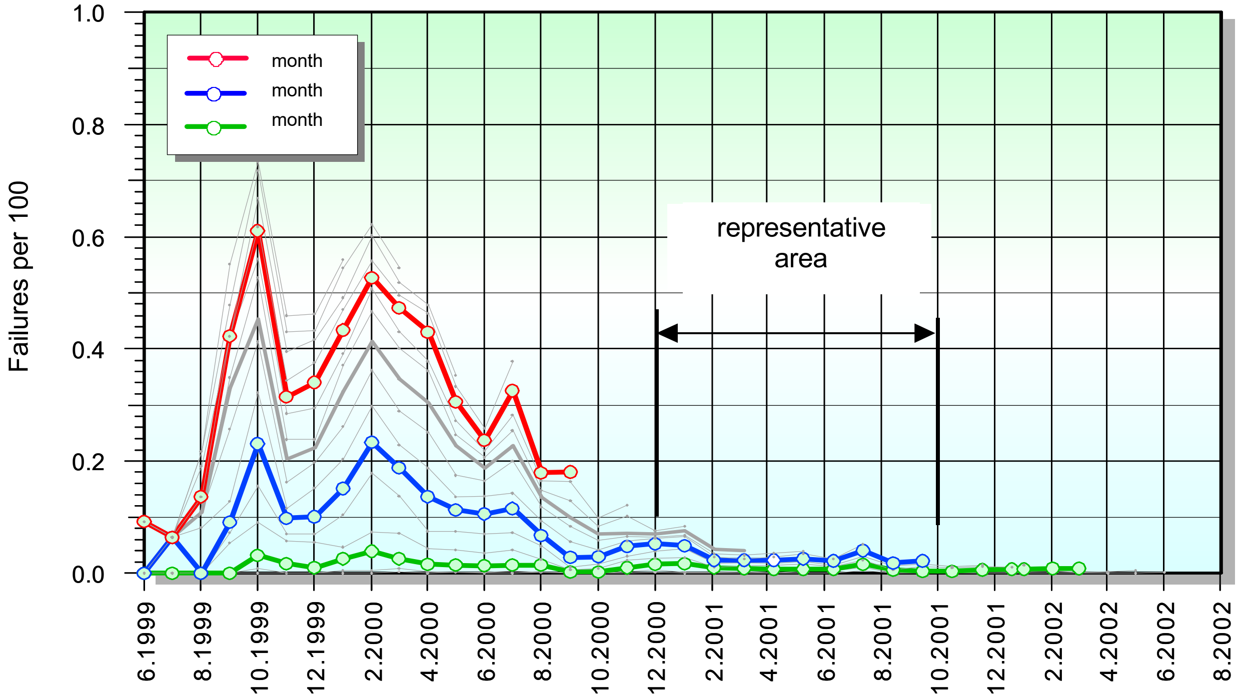
Prognosis

There are, of course, less contours the closer the production months are to the present date (because the vehicle age has not yet been reached). The number of points for determining b becomes less and less until there are only 2. Conversely, the contours can be projected for older vehicles based on the points with the youngest vehicle age together with the mean b. If, in addition to b, T^* has also been determined, the projection for any vehicle age A can be obtained through the relationship already introduced:

$$H = 100\% \; (1 - e^{-\left(\frac{A}{T^*}\right)^b})$$

The projections, however, are generally exaggerated through extrapolation of the best-fitting straight line, i.e. the values are too high. The reason for this has less to do with mathematics but rather the trend towards the higher vehicle age is digressive in the majority of cases, e.g. because of the lack of data.
Based on example, the following procedure is proposed for a prognosis. A deduction is to be made with regard to the problems expected for 10 year old vehicles.

Step 1: Definition of a representative period for today's quality situation and that expected in the future in the contour diagram:



Step 2: Transfer data section to Weibull plot and approximate linearised values with exponential function

The linearised values can be approximated with the function

$$X = \ln(t) \qquad Y = -\ln\left(\ln\left(\frac{1}{1-H}\right)\right)$$

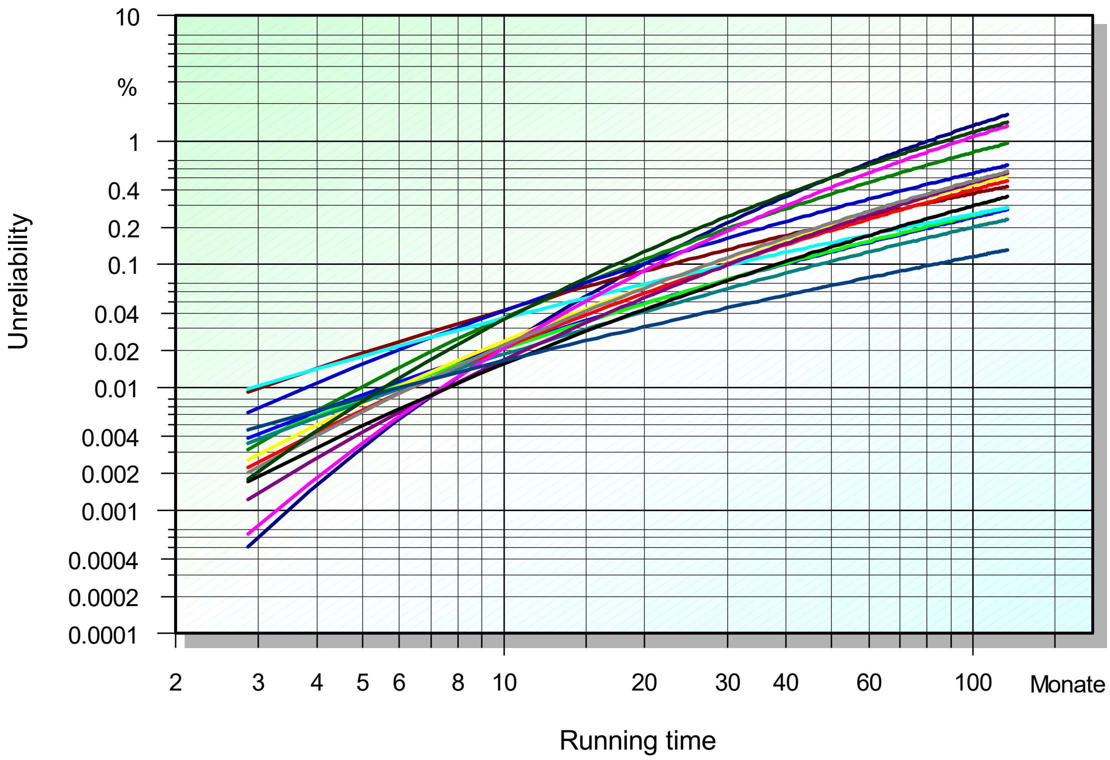
$$Y' = a' \cdot e^{b' X} \quad \text{or} \quad Y' = a' \cdot e^{b'/X}$$

In contrast to forming the best-fitting straight line, in this case, the signs of the Y-values are negated to produce positive values. However, only progressions curving to the right can be approximated with the aid of this exponential function. The Y-values must still be inverted in the case of curves with curvature to the left (progressive curve): $Y=c'-Y$. The best value for c' is generally the first linearised Y-value. The regression for determining the coefficients a' and b' then takes place with one of the above exponential functions. Accordingly, the functions for the recalculation are:

$$Y' = c' - a' \cdot e^{b' X} \quad \text{or} \quad Y' = c' - a' \cdot e^{b'/X}$$

In this case, the unit for t should be months. However, the delay must also be taken into account as the production month is specified in the contour but the date of registration is required here.

Step 3: Extrapolation with the determined function Y' in the required time range (here 10 years = 120 months). This results in a curve bundle from the extrapolations for each production month. The scatter or dispersion of the curves represents the range in which the expected problems will lie.



Conclusions can be made with regard to the lifetime of the required spare parts with the aid of the projection (long-term prognosis or forecast).

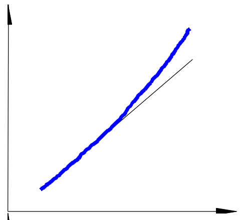
Life cycle costs (LCC)

The total costs can be estimated depending on the warranty time or based on the life cycle costs. For this purpose, the respective production figures are additionally required for each production month. An average value per complaint/problem is generally used for the costs multiplied by the number of cases. This value contains the costs for the replacement parts, the replacement and incidentals, e.g. towing costs.

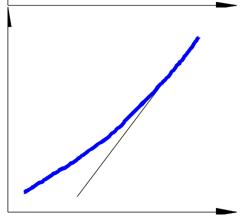
23. Appendix

Fundamental curve progressions

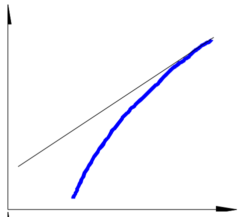
Overview of the individual representation forms and their causes



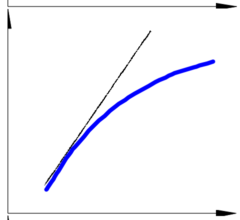
Steeper and steeper curve progression
Disregard of replacement parts (spare parts) and their shorter running times



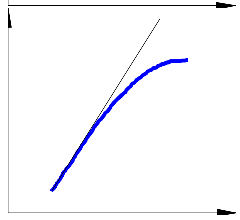
Previous damage (negative t_o)
Progression curving to the left
Defects are now always caused by the operating period. However, fault only occurs after a certain period of time.



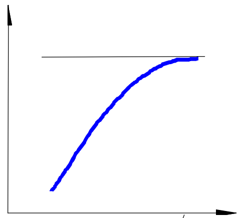
Failure-free period t_o
Progression slightly curved to the right, e.g. due to wear that becomes effective only after a certain running period



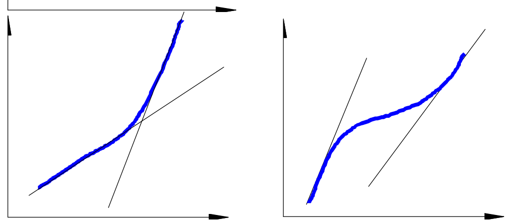
Decreasing population
Accidents and other effects decrease the number of population n (death rate). Approximately constant curvature. Handling with parameter k (3-parametric-distribution).



Lack of data
Lack of data because, for example, the warranty period has elapsed or because not all (candidates) have reached the same running time.



Subset
Only a subset of whole is affected by a failure, e.g. due to a defective batch for a limited period of time in production.



Mixed distribution
Simple or several noticeable changes in curve progression

Table of critical values for Kolmogorov-Smirnov test

The following critical values for the KS test originate from /16/. The data between 21...24 and 26...29, were interpolated.

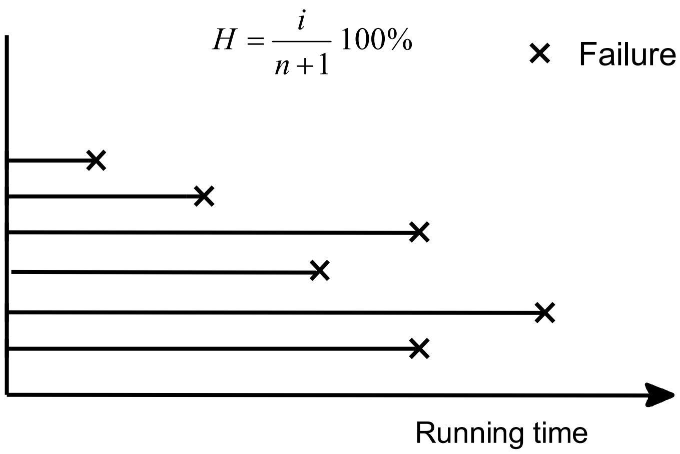
n	α =**0.1**	α =**0.05**	α =**0.01**
3	0.511	0.551	0.600
4	0.449	0.487	0.548
5	0.406	0.442	0.504
6	0.375	0.408	0.470
7	0.350	0.382	0.442
8	0.329	0.360	0.419
9	0.311	0.341	0.399
10	0.295	0.325	0.380
11	0.283	0.311	0.365
12	0.271	0.298	0.351
13	0.261	0.287	0.338
14	0.252	0.277	0.326
15	0.244	0.269	0.315
16	0.236	0.261	0.306
17	0.229	0.253	0.297
18	0.223	0.246	0.289
19	0.218	0.239	0.283
20	0.212	0.234	0.278
21	0.207	0.228	0.269
22	0.203	0.223	0.263
23	0.198	0.218	0.257
24	0.194	0.214	0.252
25	0.191	0.210	0.247
26	0.187	0.206	0.243
27	0.183	0.202	0.238
28	0.180	0.199	0.234
29	0.177	0.195	0.230
30	0.174	0.192	0.226
>30	$0{,}96/\sqrt{n}$	$1{,}06/\sqrt{n}$	$1{,}25/\sqrt{n}$

Overview of possible cases

Non-repaired units

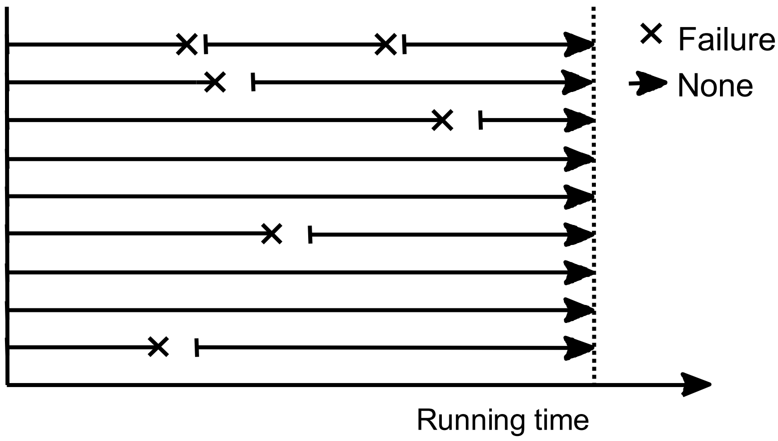
All components are used/ operated up to the point of failure. Defective components are not repaired and not further operated. This is generally the case only in connection with lifetime tests.

The quantity n required for the purpose of calculating the frequencies corresponds to the quantity of failures which is also the total number of observed units.



Repaired units

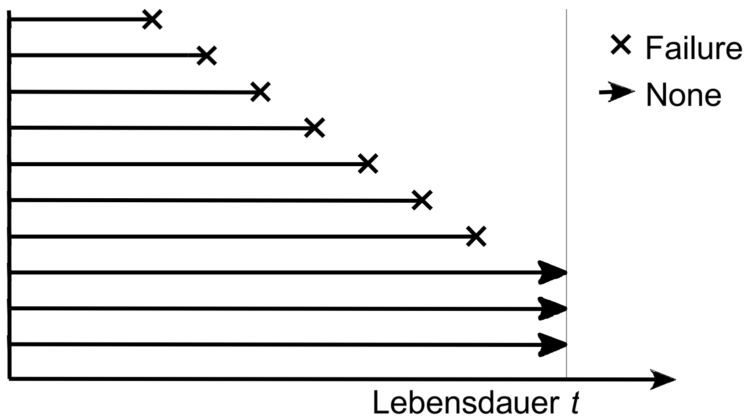
Following a failure, it must be possible to continue to use units that are in use/operation. This means defective components are replaced. In this case, it is necessary to take into account only the actual running times of the failures (referred to zero line). The calculation then takes place as described in the above.



The quantity n required for the frequencies corresponds to the number of units including replacements. The total quantity originally produced therefore increases by the number of replacement parts.

Incomplete data
Simple case:

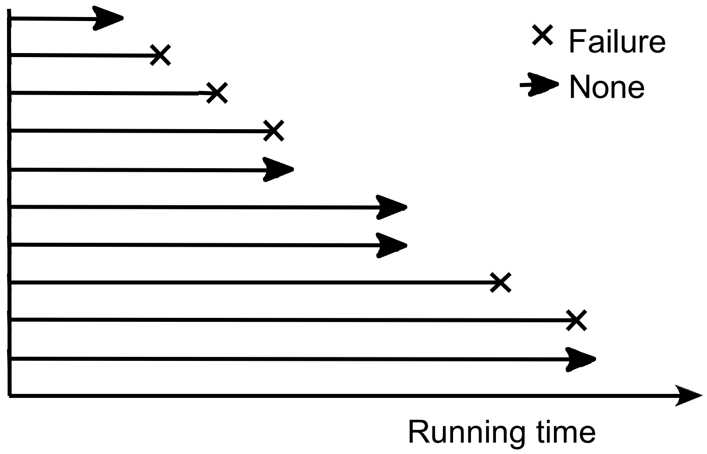
All parts that have not failed have the same operating performance rating (mileage).
The quantity n required for the frequencies corresponds to the number of failures plus the units still in use/ operation (= total number).

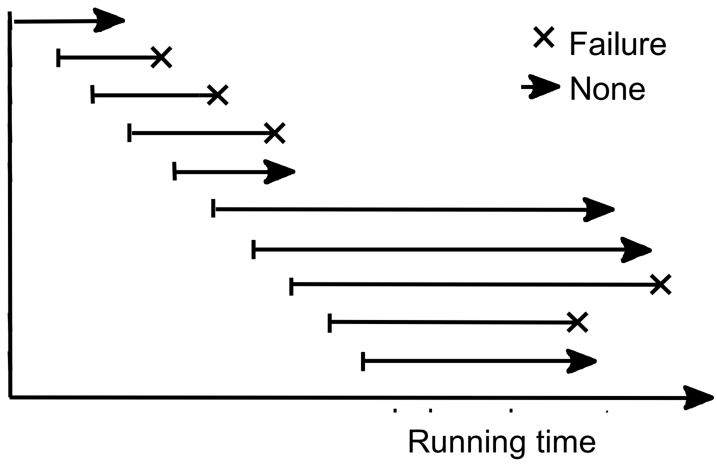


Running time

General case:

This case involves failures and parts with different running times. Special calculation methods are required for this purpose that will be explained later.
Another possibility is that the units are used at different starting points as is the case, for example, in current series production.
The quantity n required for the frequencies corresponds to the number of failures plus the parts still running.





Overview of distributions

Frequency distribution / Histogram

A frequency distribution shows the frequency of identical values. Let us assume the values listed in column A represent the diameter of a rotating shaft. All identical values are counted and the frequencies entered in the adjacent column B.

A	B
9.98	1
9.99	
9.99	2
10	
10	
10	3
10.01	
10.01	2
10.02	1

⇨

A	B
9.98	1
9.99	2
10.00	3
10.01	2
10.02	1

The values are combined to give on the right table:

The mean value $\bar{x}$ is calculated using $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

and the standard deviation s with $s = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

where n represents the amount of data. With these data it is possible to determine the so-called Gaussian or normal distribution that is represented as a curve (bell curve). Great importance is attached to the normal distribution in practical applications. It represents the mathematically idealised limit case which always occurs when many independent random influences are added.

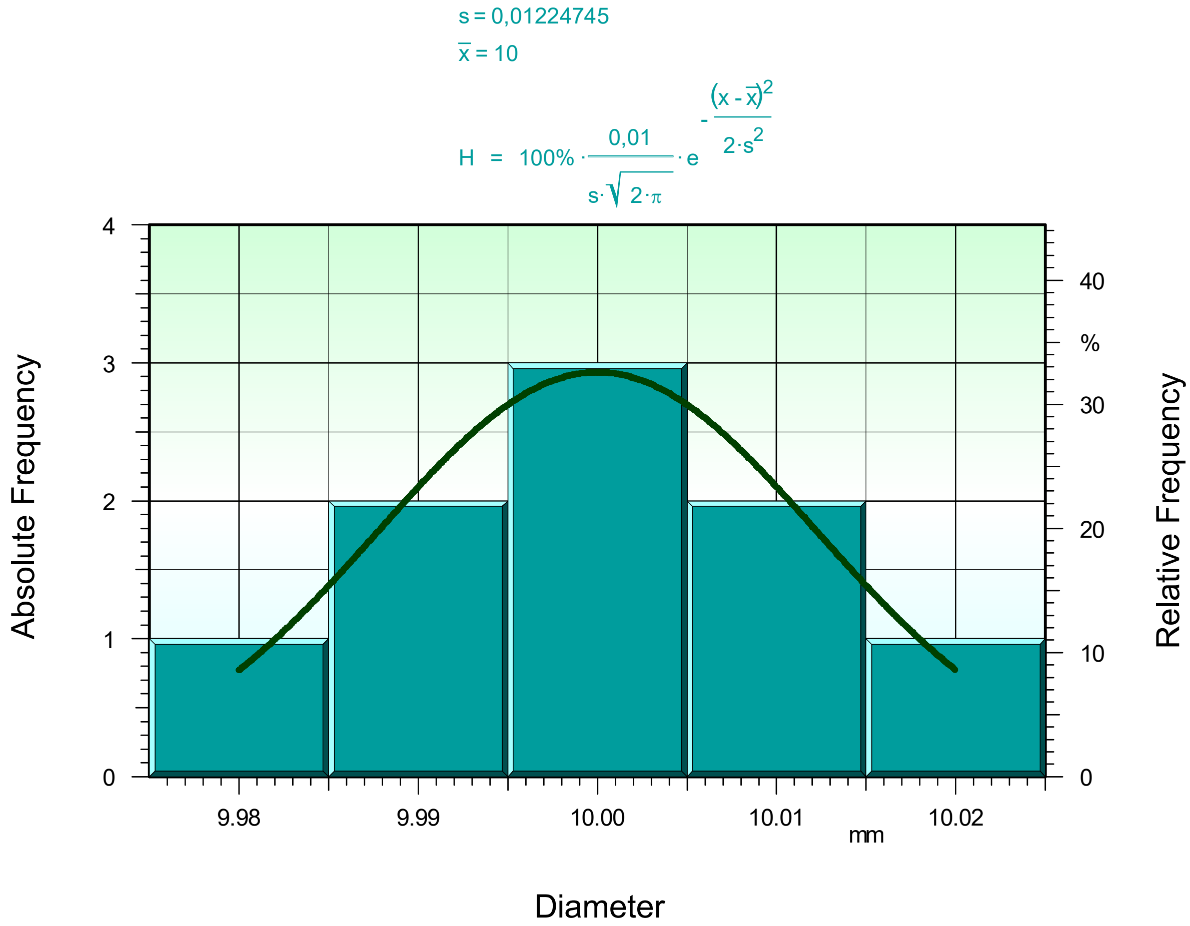
The general density function is:

$H = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(x-\bar{x})^2}{2s^2}}$ and for classified data $H = K \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(x-\bar{x})^2}{2s^2}}$

were

- H : Frequency (standardised to 1 in % times 100)
- s : Standard deviation
- $\bar{x}$: Mean
- K : Class width

For the approximation of class data, it is necessary to extend the density function by the class width so as to correctly take into account the corresponding individual frequency, referred to the units.



The data must be sorted in ascending order for representation purposes. Series of measurements with data lying very close together are often encountered in practical applications. Parameters with exactly the same value occur only very rarely or not at all. The frequency distribution would therefore determine each parameter only once. In such cases, classification is used, i.e. ranges are defined within which data are located, thus improving the frequencies. The classification is based on the formula:

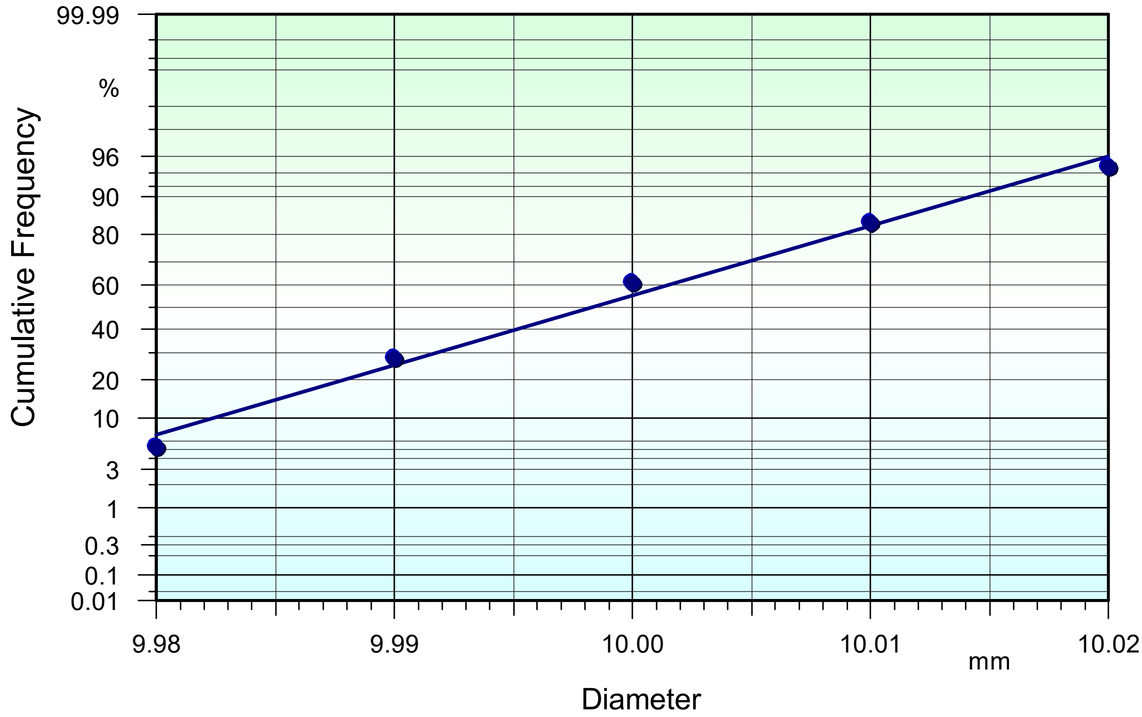
Value = rounding-off (value/class width)

Cumulative frequency – Normal distribution

The cumulative frequency also known as the probability plot represents the sum of the frequencies from the lowest value up to the considered point x. The cumulative curve is the integral of the density function. The normal distribution is expressed by the formula:

$$H = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(x-\bar{x})^2}{2s^2}}$$

Concrete values are applied in terms of their frequencies above of the associated upper class limits as a sum total or cumulative value (see frequency distribution for explanation of classes). The values entered for the example from the frequency distribution appear in the probability plot as follows:



Compared to the frequency distribution, this representation offers the advantage that it is easy to read off the percentage of the measured values within each interval (estimation of percentage of failures outside the tolerance. In addition, it is very easily possible to show how well the values are normally distributed, i.e. when they are as close as possible or preferably on the cumulative curve.

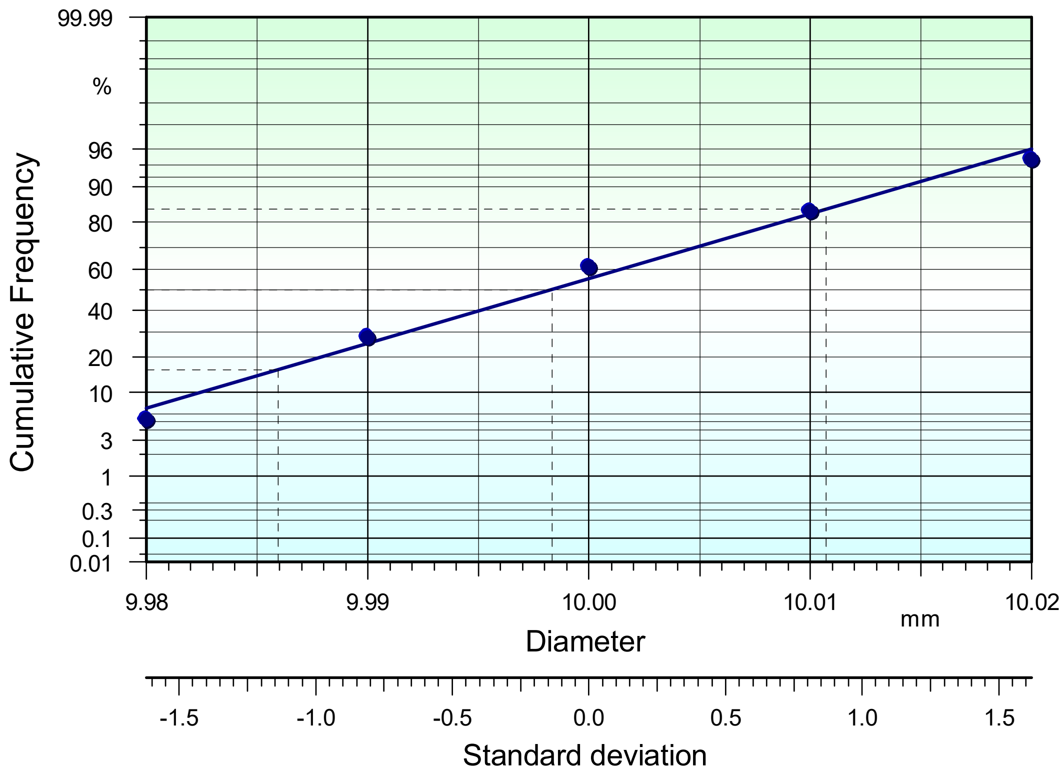
The frequencies in the probability plot are defined by /23/:

$$H = \frac{i - 0{,}535206}{n - 0{,}070413} \cdot 100\%$$ where i = Ordinal of the sorted values

or by approximation with:

$$H = \frac{2i - 1}{2n} \cdot 100\%$$

Note: The cumulative frequencies given by these equations do not result exactly in the cumulative individual frequencies as they are referred to probabilities in this case.
An S-shaped cumulative curve is normally obtained between the points. The straight line obtained in this case is due to the fact that the ordinates have been correspondingly distorted logarithmically.
The mean (here $\bar{x}$ = 10.0) coincides exactly with the cumulative frequency of 50%.
The range of $\bar{x} \pm s$ is located at 16% and 84% frequency.
In practical applications, the cumulative frequency is often represented relative to the scatter ranges of ±1s, ±2s and ±3s. This simply means that the X-axis is scaled to the value of s and the mean is set to 0.



Log-normal distribution

The log-normal distribution is a distribution that is distorted on one side and exhibits only positive values. A graphic illustration that a feature is not distributed symmetrically and that the feature cannot undershoot or overshoot a certain bound. A good example is the distribution of times that cannot be negative. Particularly when the distribution is limited to the left by the value 0, approximately normal distribution values can be achieved by taking the logarithm. The creation of a log-normal distribution may also be attributed to the fact that many random variables interact multiplicatively.
The failure characteristics of components in terms of the classic operating strength (e.g. fatigue strength and reverse bending stresses and cracking/fracture fault symptoms), are generally best described through the log-normal distribution. In addition, the distributions of distances covered by vehicles are generally defined by log-normal distribution.
The cumulative curve is the integral of the probability density. The log-normal distribution is expressed by:

$$H = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi s^2}} \frac{1}{x} e^{-\frac{(\ln(x)-\bar{x})^2}{2s^2}}$$

Unlike many other distributions, the log-normal distribution is not included as a special case in the Weibull distribution. However, it can be approximated using the 3-parameter Weibull distribution.

The log-normal distribution such as the cumulative frequency is represented by the integral of the density function. Instead of the mean and the standard deviation, the median and the dispersion coefficient are of significance in connection with the log-normal distribution. The median is derived through the perpendicular of the point of intersection of the 50% cumulative frequency with the fitting line on the X-axis or analytically through:

$$\log x_{50\%} = \frac{1}{n}\left(\sum_{i=1}^{n}\log(x[i])\right)$$

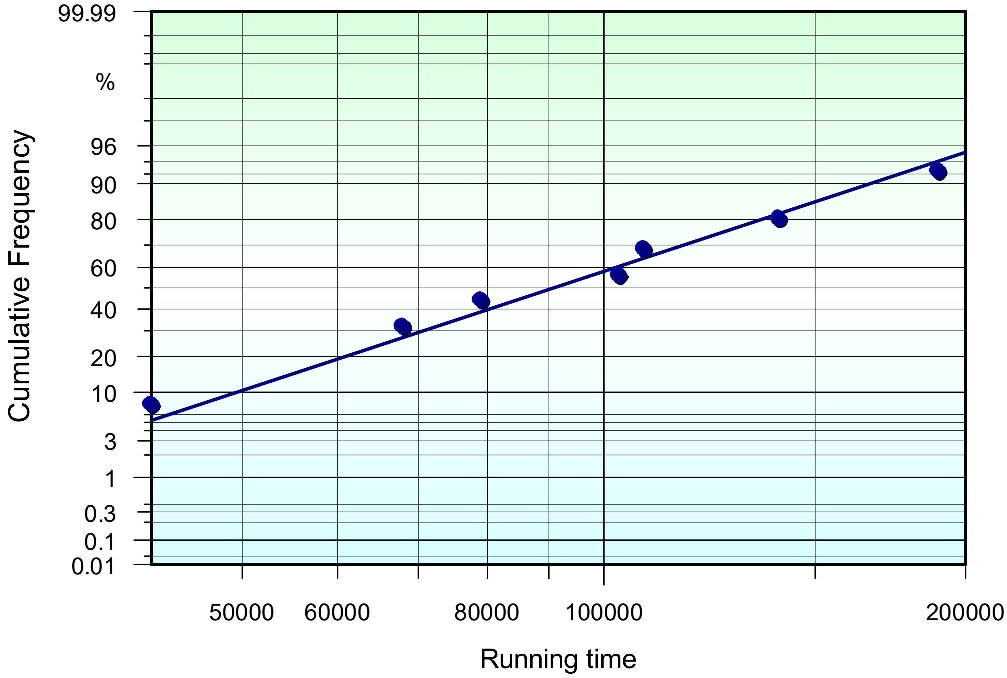
Median = $10^{\log x_{50\%}}$

Log. standard deviation

$$s_{\log} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\log(x_i)-\log(x_{50\%})\right)^2}$$

Dispersion factor = $10^{s \log}$

The points of intersection with the 16% and 84% cumulative frequency do not correspond to the range for $\bar{x} \pm$ s as for the "normal" cumulative frequency, but rather they correspond to the range for *median / dispersion factor* and *median * dispersion factor*.



The range between 10% and 90% is often represented instead of 16% and 84%.
This is derived from:

$$x_{10\%} = x_{50\%} \; / \; 10^{1{,}28155 \cdot s_{\log}} \quad \text{and} \quad x_{90\%} = x_{50\%} \cdot 10^{1{,}28155 \cdot s_{\log}}$$

where 1.28155 is the quantile of the standard normal distribution for 10%.

When determining the straight line analytically, it is derived only from the median and the dispersion factor. Visually, the points may in part lie on one side depending on the frequency values.

These deviations can be reduced by implementing a Hück correction factor

$$k = \sqrt{\frac{n-0{,}41}{n-1}}$$

$$x'_{10\%} = x_{50\%} \; / \; 10^{1{,}28155 \, k \cdot s_{\log}} \quad \text{and} \quad x'_{90\%} = x_{50\%} \cdot 10^{1{,}28155 \cdot k \cdot s_{\log}}$$

As a result, the straight line becomes correspondingly flatter.
The frequencies of the individual points are recommended in accordance with Rossow:

$H = \frac{3i-1}{3n+1} \cdot 100\%$ for $n \leq 6$ and $H = \frac{i-0{,}375}{n+0{,}25} \cdot 100\%$ for $n > 6$

where i = Ordinal of the sorted X-values

If the frequencies are already defined in percent, the straight line can only be determined using the method of the fitting line with linearised points.

Weibull function

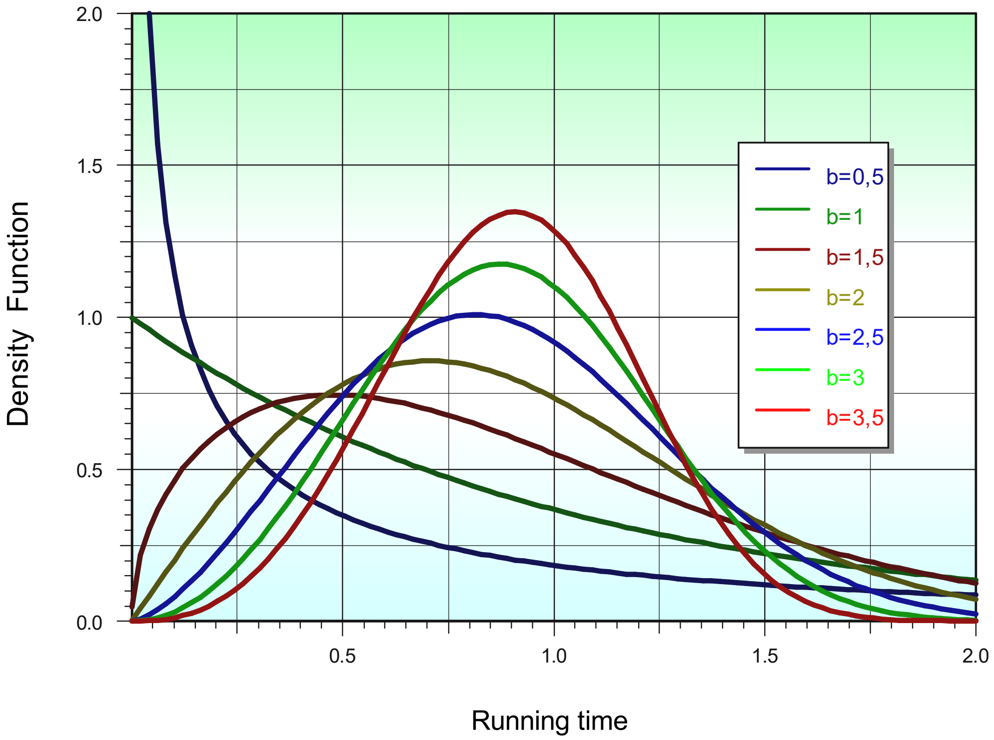
The density function of the Weibull distribution is represented by:

$$h = \frac{b}{T}\left(\frac{t}{T}\right)^{b-1} \cdot e^{-\left(\frac{t}{T}\right)^{b}}$$

were

h = Probability density for "moment" t
t = Lifetime variable (distance covered, operating time, load or stress reversal etc.)
T = Scale parameter, characteristic life during which a total of 63.2% of the units have failed
b = Shape parameter, slope of the fitting line in the Weibull plot

The following curve is obtained for various values of the shape parameter b and a scaled T=1:

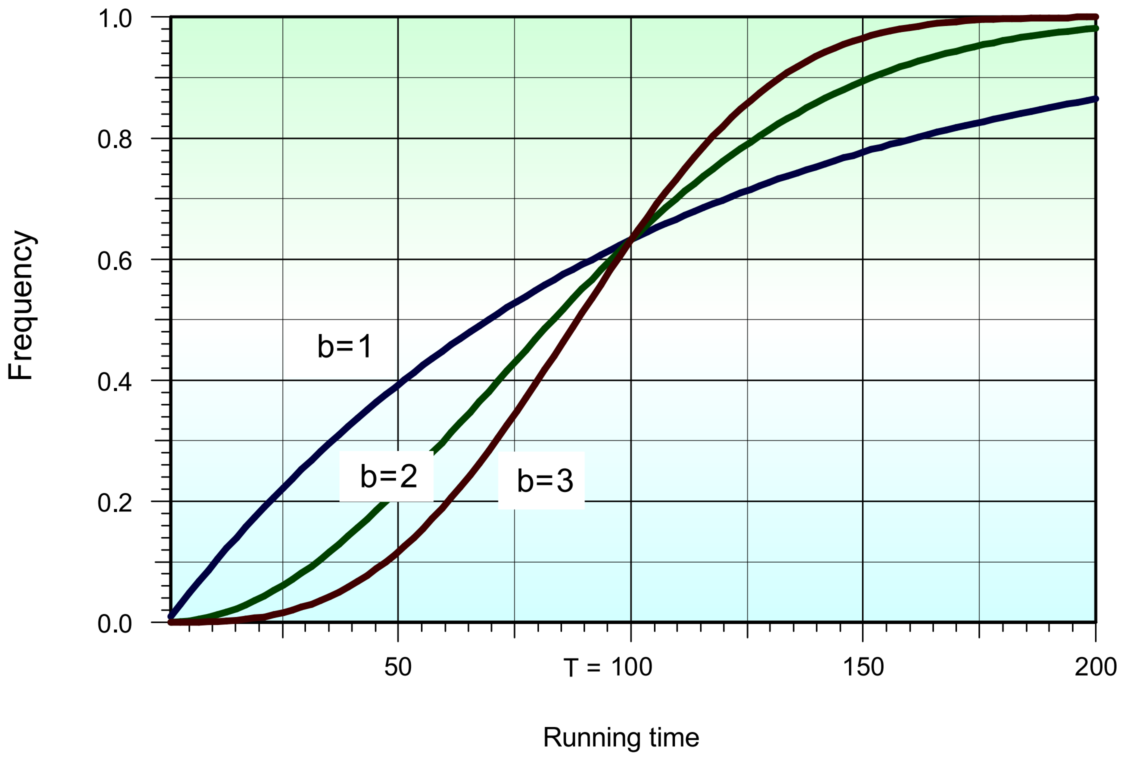


Great importance is attached to the cumulative frequency or the integral of the density function which expresses the so-called failure probability. With this function it is possible to determine how many parts have failed or will fail up to a defined running time.
When represented in a linear diagram, an S-shaped line results over the entire progression which is not easy to read off. In its simplified 2-parameter form (see /1/ and /2/) the Weibull distribution function is:

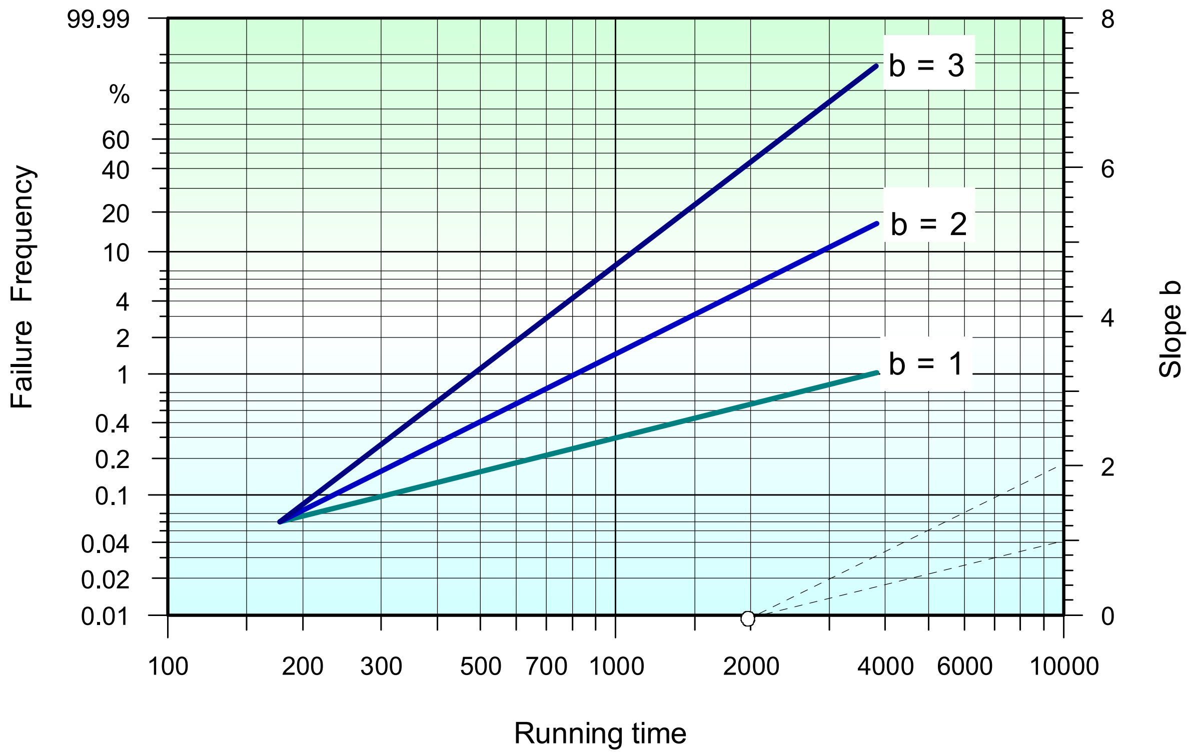
$$H = 1 - e^{-\left(\frac{t}{T}\right)^b}$$

were

H = Cumulative failure probability or failure frequency
(scaled to 1, in % times 100)



The S-shaped line is made into a straight line linearised best-fit straight line) by the distortion of the ordinate scale (double logarithmic) and of the abscissa (logarithmic).The advantage of this is that it is easy to recognise whether the distribution is a Weibull distribution or not. In addition, it is also easier to read off the values. The slope of the straight line is defined as a direct function of the shape parameter b. For this reason, an additional scale for b is often represented on the right. The slope can be determined graphically by shifting the straight line parallel through the "pole“ (here at 2000 on the X-axis).



There is also the 3-parameter form

$$H = 1 - e^{-\left(\frac{t - to}{T - to}\right)^b}$$

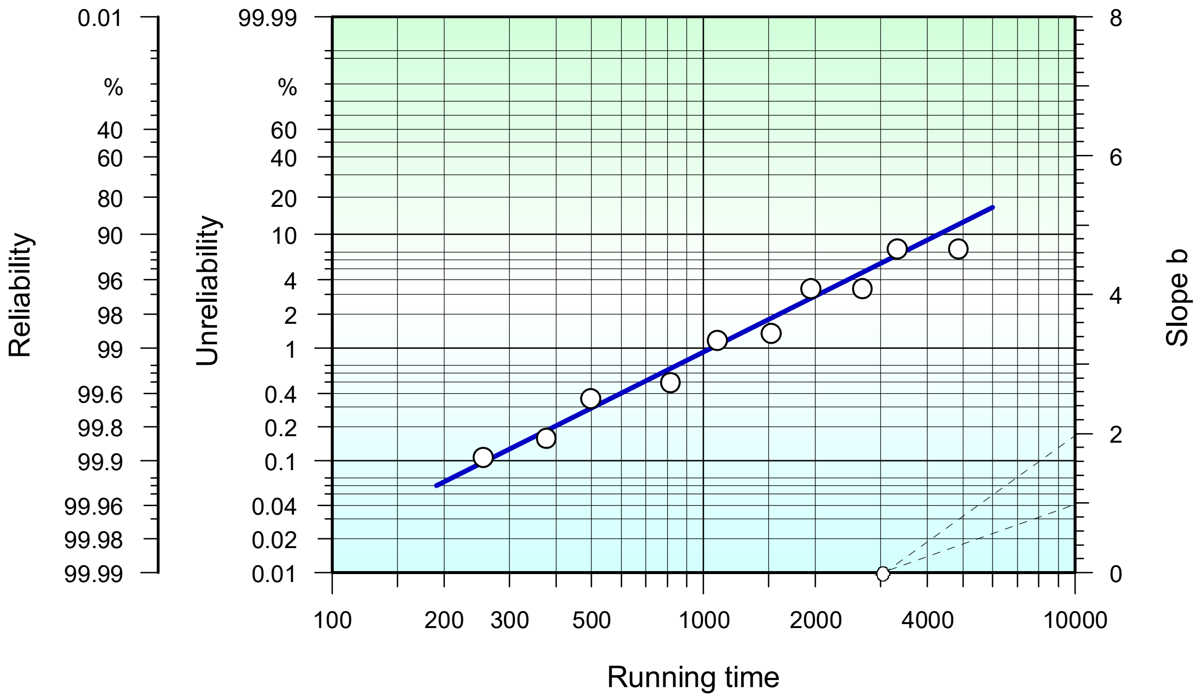
where t_o = time free of failures

In the majority of cases it is possible to calculate with t_o = 0 what the 2-shape parameter corresponds to. Despite being subject to stress load, some components behave such that failures occur only after an operating time t_o . In connection with this behaviour, the points above the lifetime characteristic are mostly curved to the right in the Weibull plot. In the case of the curve dropping steeply to the left, with t_o it is possible to imaging the point of intersection of the curve with the zero line which is in infinity on the logarithmic scale. The procedure for determining the time t_o free of failures it is discussed in a separate chapter.

The so-called reliability is often used instead of the failure frequency:

$$R = e^{-\left(\frac{t}{T}\right)^b} \quad \text{or} \quad R = 1 - H$$

It indicates how many parts are still in use after a certain running time and therefore have not yet failed. The Y-axis in the Weibull plot extends from top to bottom:



If failure frequencies are low, the specification ppm (parts per million) is also appropriate instead of the percentage. In this case 1% = 10000 ppm.

Beta

Supplies values of the distribution function for random variables with beta distribution. The distribution function (integrated density function) of a beta distribution is used to examine percentage fluctuations over several random samples taken at certain procedures. For example, it is possible to examine what percentage of a day certain people sit in front of the television. Further applications include processes with natural upper and lower limits. The probability distribution is:

$$h = \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}$$

x : Variable ($0 \le x \le 1$)
α : Is a parameter of the distribution ($1 \le \alpha$)
β : Is a parameter of the distribution ($1 \le \beta$)

Binomial

The binomial distribution describes the number of defective units in random samples which are "put back". The number of defective units in the random sample can be used to monitor the share of defective units in the population. The probability density is:

$$h = \binom{n}{x} p^x (1-p)^{n-x}$$

x : Variable
n : Scope
p : Relative share of defective parts

Cauchy

The probability density is:

$$h = \frac{1}{\theta\pi\left(1+\left(\frac{x-\eta}{\theta}\right)^2\right)}$$

x : Variable
η = Median position parameter
Θ = Theta scaling parameter

χ^2 (Chi²)

Supplies the values of the distribution function (1-α) of a random variable with χ^2-distribution. The χ^2-distribution is required in connection with a χ^2-test. The χ^2-test can be used to compare observed and expected values.
For example, the hypothesis can be proposed for a genetic experiment that the next plant generation will have a certain colour configuration. By comparing the observed and the expected results it is possible to determine whether the original hypothesis is correct. The probability density is:

$$h = \frac{1}{2^{\nu/2}\Gamma(\nu/2)}\left(x^{\nu/2-1}e^{-x/2}\right)$$

x : Variable x > 0
ν : Number of degrees of freedom

Exponential

Supplies the probability of a random variable with exponential distribution. This distribution is used for the purpose of calculating the probability that a procedure requires a certain time between two events. A possible question, for example, could be: How long does a cash dispenser (ATM) require to issue money? You could, for example, calculate what is the probability that this procedure takes one minute. The probability density is:

$$h = \lambda e^{-\lambda x}$$

x :Variable (x >= 0)
λ : is the parameter of the distribution (lambda > 0)

Extreme

The extreme distribution is often used for the purpose of modelling extreme events such as the extent of flooding, wind speeds aircraft are confronted with, maxima of share indices etc. A further application involves reliability theory, e.g. representing the distribution of the failure times for electrical power circuits. The probability density is:

$$h = \frac{1}{b}e^{-(x-\theta)/b}e^{-e^{-(x-\theta)/b}}$$

x : Variable
a : Positional parameter
b : Scaling parameter (b>0)

Fisher

The Fisher or F-distribution supplies values for the distribution function (1-alpha) of a random variable. With this function it is possible to determine whether two data sets have different dispersions. For example, it is possible to examine the scores men and women achieved in a university acceptance test and consequently determine whether the scatter or dispersion found for women differs from that of men.
If the variances s_1^2 and s_2^2 of independent random samples of the range n1 and n2 are from two populations with normal distribution and the same variance, the random variable will be

$$F = \frac{s_1^2}{s_2^2}$$

of an F-distribution with the degrees of freedom $\nu = n_1 - 1$ and $\omega = n_2 - 1$.

The F-distribution is a consistently asymmetric distribution with a variation range from 0 to infinity. The probability density is:

$$h = \frac{\Gamma((\nu+\omega)/2)(\nu/\omega)^{\nu/2} x^{(\nu-2)/2}}{\Gamma(\nu/2)\Gamma(\omega/2)(1+(\nu/\omega)x)^{(\nu+\omega)/2}}$$

x : Variable($x \geq 0$)
ν , ω : Number of degrees of freedom

Gamma

Supplies the probabilities of a random variable with gamma distribution. With this function it is possible to examine variables that have a skewed distribution. The gamma distribution is often used for the purpose of analysing queues. The probability density is:

$$h = \frac{1}{b\Gamma c}\left(\left(\frac{x}{b}\right)^{c-1} e^{-x/b}\right)$$

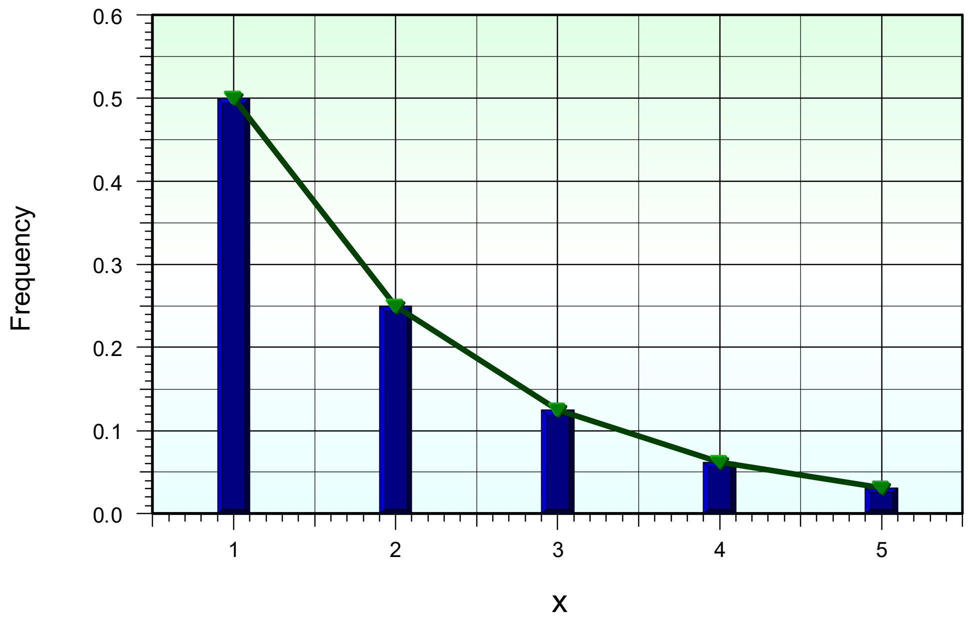
x : Variable ($x \geq 0$)
α : Is a parameter of the distribution ($\alpha > 0$)
β : Is a parameter of the distribution ($\beta > 0$)

Geometric

The geometric distribution is used for establishing, for example, the service life of a device up to the first time a fault occurs or the period of time a certain customer files a claim for the first time with his/her motor vehicle insurance or the time or unemployment until a person is employed again. The geometric distribution is a discrete distribution, i.e. it applies only to integer arguments (x). The probability density is:

$$h = p(1-p)^{x-1}$$

x : Variable, integer
p : Probability of a favourable event ($0 < p < 1$)



Hypergeometric

The hypergeometric distribution is used instead of the binomial distribution when samples are taken (without putting back). This distribution is often used in connection with problems concerning quality monitoring. The hypergeometric distribution is a discrete distribution, i.e. it applies only to integer arguments. The probability density is:

$$h = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

x : Variable, the number of successes achieved in the sample
N : The scope (size) of the population
M : The number of successes possible in the population
N : The scope (size) of the sample
All parameters are integers

Laplace

The probability density is:

$$h = \frac{1}{2b} e^{-|x-a|/b}$$

x : Variable
a : Mean
b : Scaling parameter

Logistic

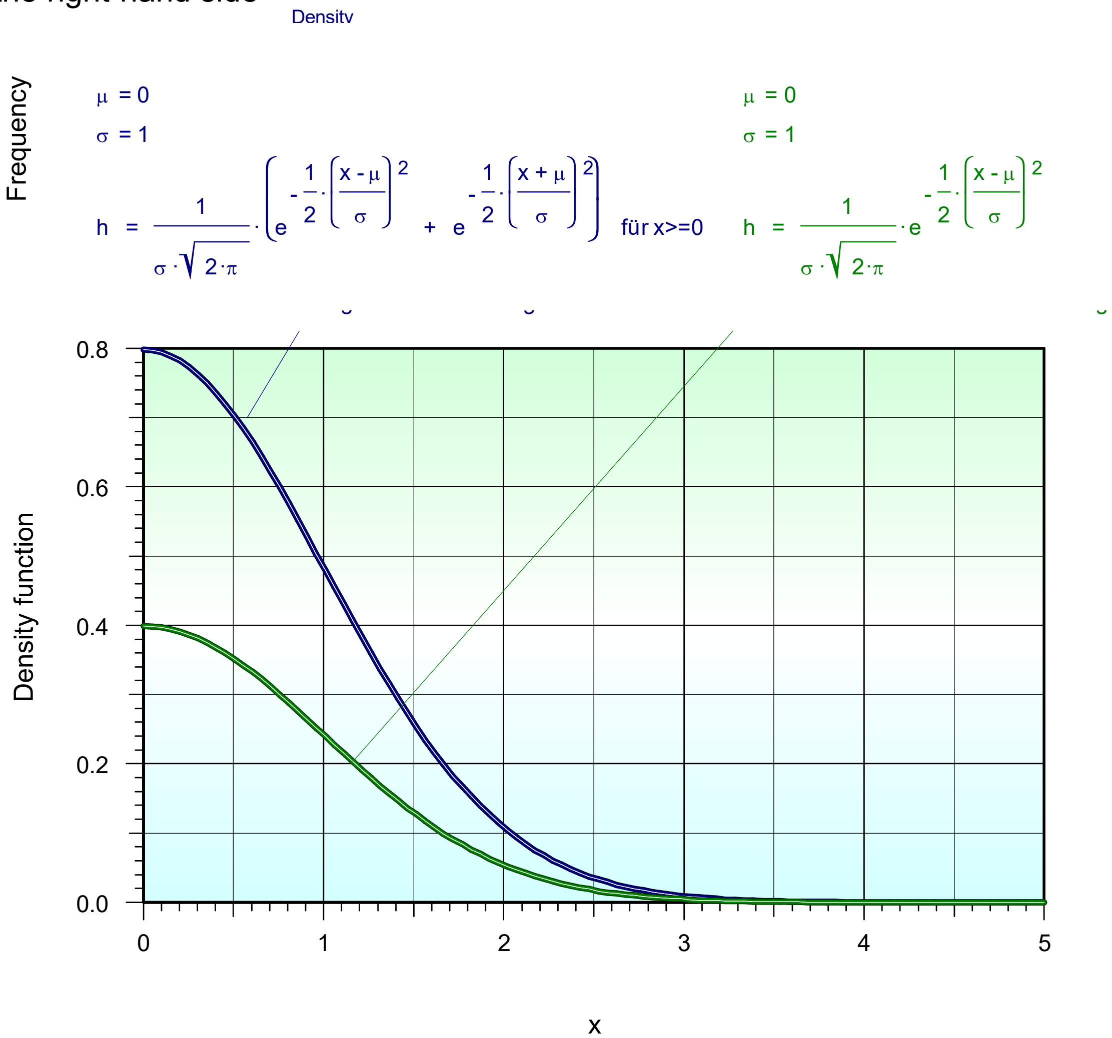
The probability density is:

$$h = \frac{\frac{1}{b} e^{-(x-a)/b}}{\left(1 + e^{-(x-a)/b}\right)^2}$$

x : Variable
a : Mean
b : Scaling parameter

Truncated or folded normal distribution

The so-called truncated normal distribution results by folding the negative half onto the right-hand side



Pareto

The probability density is:

$$h = \frac{c}{x^{c+1}}$$

x : Variable ($x \geq 0$)
c = Parameter ($c < 0$)

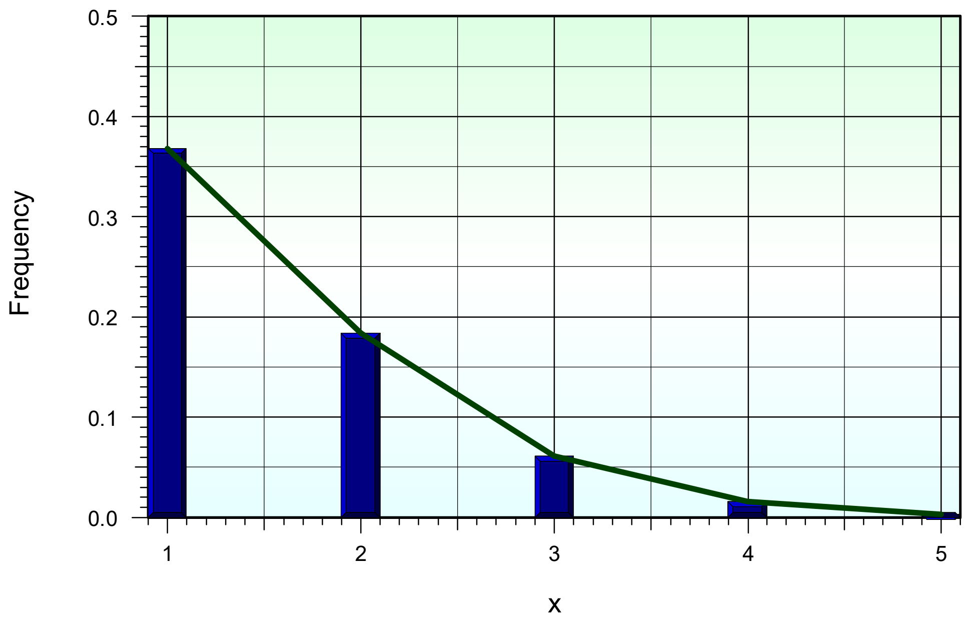
Poisson

The Poisson distribution describes the number of defects in random samples when the defects occur independently. Several defects in a unit are possible. The number of defects in the random sample can be used for the purpose of monitoring the mean number of defects per unit in the population. The Poisson distribution is a discrete distribution, i.e. it applies only to integer arguments (x). The probability density is:

$$h = \frac{\lambda^x e^{-\lambda}}{x!}$$

x : Variable, integer

λ : Mean (λ>0)



Rayleigh

The Rayleigh distribution is used, for example, in connection with offsets or eccentricities. The probability density is::

$$h = \frac{x-\theta}{b^2} e^{-((x-\theta)^2/(2b^2))}$$

x : Variable (x > Θ)
b = Scaling parameter (b > 0)
Θ = Positional parameter

Student

The t-distribution is very similar to the normal distribution. However, it does not depend on μ and σ. The shape of the t-distribution is determined only by the degree of freedom ν=n-1. The t-distribution approaches the normal distribution the greater ν is. The main application of this type of distribution is the comparison of mean values of various samples. The probability density is:

$$h = \frac{\Gamma((\nu+1)}{2\Gamma(\nu/2)} (\nu\pi)^{-0.5} (1+(x^2/\nu))^{-(\nu+1)/2}$$

x : Variable
ν : Degree of freedom

Symbols used in formulae

Variable	Definition	Description
A	Age/vehicle age	Or operating time

A_D	Availability	Also permanent availability of components
b	Shape parameter in Weibull	Slope of the best-fitting straight line in the Weibull plot
f	Degree of freedom	For statistical tests
Fk	Corrected failure cumulative frequency	Prognosis calculation
Fan	Candidates, cumulative frequency	Prognosis calculation
H	Generally frequency or failure frequency	Mostly in %
i	Ordinal	Generally: consecutive index
k	Number of classes	
k	Slope in the stress-cycle (Woehler) diagram	
K_{br}	Class width	
$LproM$	Distance (mileage) per month	
L_v	Service life ratio	Referred to required service life
λ_T	Failure rate	
$MTTF$	Mean time-to-failure	Corresponding to expectation value t_m
MTBF	Mean operating time between failures	
$MTTR$	Mean time-to-repair	
N	Number of alternating stress cycles	Woehler
N_D	Number of alternating stress cycles in which fatigue strength begins	Woehler
n	Number of defects, parts, random samples or degrees of freedom	Generally: Number of parts
Nan	Number of candidates	Prognosis calculation
$\overline{x}$	Generally mean of sample	$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Δx	Class width	Generally: Step width
μ	Mean of population	
p	Success probability	
P_A	Confidence level	
R	Range	$R = x_{max} - x_{min}$
R	Reliability	1-H
s	Standard deviation	Special form for Weibull
s^2	Variance of sample	Special form for Weibull

σ	Standard deviation of population	Special form for Weibull
σ^2	Variance of population	Special form for Weibull
σ	Component stress	Woehler
σ_D	Component stress, from which fatigue strength occurs	Woehler
t	Life variable in Weibull	Distance, operating time, alternating stress cycle etc.
t_m	Expectation value/mean for Weibull	
t_o	Failure-free period	
T	Characteristic life in km or alternating stress cycles	For 63.2% failure frequency
T^*	Characteristic life in months	For 63.2% failure frequency
V	Confidence bound	
w	Weighting	Number of defined value
α	Significance level for statistical testing	The transfer parameter alpha is alpha = 1- α or 1 - α/2 for two-sided tests
X	Variable X diagram	
$X1..X3$	Reference point	For distance (mileage) distribution
Y	Variable Y diagram ordinate	

24.Most important formulas

Expected life time Mean Time to Failure *MTTF*	$t_m = (T - t_o)\Gamma\left(1 + \frac{1}{b}\right) + t_o$
t_{10}- (B_{10}) Life time	$t_{10} = (T - t_o) \cdot 0{,}1054^{\frac{1}{b}} + t_o$
Median 50% probability	$t_{50} = (T - t_o) \cdot 0{,}6931^{\frac{1}{b}} + t_o$
Life time for defined unreliability (H normed to 1)	$t = (T - t_o)\left(\ln\left(\frac{1}{1-H}\right)\right)^{\frac{1}{b}} + t_o$
Failure rate (relative number of faults in the next interval, often defined per hour)	$\lambda = \frac{h_{(t)}}{1 - H_{(t)}} = \frac{b}{T - t_o}\left(\frac{t - t_o}{T - t_o}\right)^{b-1}$

25. Literature

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